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The International Journal of Mathematics and Consciousness is a semi-annual journal dedicated to the mathematical description and understanding of consciousness. Address correspondence to: *International Journal of Mathematics and Consciousness*, Department of Mathematics, Maharishi University of Management, Fairfield, Iowa, 52556, USA; email IJMAC@mum.edu.

EDITORIAL INTRODUCTION TO THE JOURNAL

Throughout history, natural phenomena have been ultimately mysterious. Some of these phenomena were explained by religious belief, others by philosophical analysis. Since the 17th century, the modern scientific approach has found that many phenomena in nature obey clearly describable physical laws. This success greatly widened the ambit of scientific inquiry beyond the physical into the realm of what previously had been considered metaphysical or nonmaterial. Today, the territory of scientific inquiry has expanded to include how matter leads to consciousness.

Most common and popular models of consciousness share the postulate that physical activity in the brain is prior to consciousness. No current theory, however, has been able to resolve the problem of how physical processes in the brain give rise to subjective experiences. Even quantum mechanical theories, while suggesting potential mechanisms that might create "unexplainable" phenomena, fall short of answering the fundamental questions about subjective experience. This gap—between the objective, material brain and the intimately known, private qualia of subjective experience, or "what it is like" to experience something—has so far not been bridged. Some thinkers have even rejected qualia out of hand, asserting that we have insufficient knowledge of the physical world to evaluate their existence.

Some believe that early *Homo sapiens* depended entirely on sensory experience as a reference for what does and does not exist, and that only as our understanding evolved did we come to challenge the evidence of our senses. Certainly, the discoveries of modern science changed the way we looked at the world. They gave us intellectual models of the universe that often seemed to contradict our sensory model but which provided in fact more accurate pictures and were eventually confirmed by experimental observation.

Perhaps the most notable example is the shift from a geocentric to a heliocentric view of the cosmos as a result of the work of Copernicus, Kepler, and Galileo in the 16th and 17th centuries. More recently, inquiry into very small and very large time and distance scales in relativity theory, quantum mechanics, quantum field theory, and cosmology has radically changed our beliefs about the nature of matter and physical phenomena as our senses perceive and our intellects apprehend them. We may ask, what actually exists for us? And we may agree that everything is continuously changing; we may even agree that whatever appears not to change is only one of an infinite number of simultaneously existing possibilities. For example, in some models a particle can be everywhere at once, and the fact that we find it here and now suggests either that we have collapsed the infinitude of its possibilities in a single act of conscious experience or that it continues to exist everywhere in an infinite number of universes parallel to the one in which we experience it.

In all this uncertainty, one fact seems undeniable: the fact of our own awareness. Without awareness, we can neither perceive nor apprehend, neither see nor think nor dream. Commonly, this awareness is called consciousness: the observer, the witness, the experiencer. If indeed this is the one undeniable fact, then it is timely that a scientific journal be dedicated to the study of consciousness as primary.

To be truly scientific requires that the journal obey rigorous methods of logic, research, and experimentation. At the same time, this requires that no *a priori* or unproven points of view stand in the way of new original postulates, previously explored theories revisited with new insights, or unconventional axioms.

The *International Journal of Mathematics and Consciousness* is founded in part to fulfill this need. The *Journal* opens the door to all mathematicians, scientists, and thinkers to present their theories of consciousness and the consequences thereof. With the requirement that such theories follow strict mathematical, logical argumentation and respect proven facts and observations, articles can be submitted for review, without restriction to their proposed axioms and postulates. The *Journal* also welcomes carefully reasoned articles that challenge commonly held, but not fully established, theories and beliefs.

1. Consciousness and "Consciousness at work"

Abstract concepts and subjective experiences such as love, friendship, beauty, devotion, happiness, inspiration, pain, despair, and deception, are, in and by themselves, hard to study scientifically because of their innate, subjective, personal nature. Even more difficult to study is the more abstract consciousness that seems to be like a screen on which these emotions, notions, and sensations are projected and experienced.

Modern cognitive neuroscience identifies various neural correlates of these mental states. The discipline of psychology attracted great thinkers who proposed various theories and methods of investigation, mostly focusing on the manifestations, observable or subjectively reportable signs and symptoms, and causes and effects of such inner experiences. Physicists recently have attempted to bridge the gap between the physical world and conscious experience through various quantum mechanical models.

Philosophy, metaphysics, and spiritual and religious studies delve into ontological, epistemological, and other fundamental questions, using more or less formal logic or a wide variety of opinions and postulates. In contrast, art forms such as music, painting, and fictional writing are outer expressions of inner experiences and creative thinking.

All theories, concepts, and creative work, whether scientific, psychological, philosophical, artistic, or spiritual are the manifestations of "consciousness at work." While it might be challenging to study "consciousness" as such, in and by itself, it may be easier to study "consciousness at work"—its dynamics and its manifestations.

The postulates that can be made about consciousness as an abstract phenomenon or epiphenomenon are most amenable to investigation by scientifically analyzing and studying "consciousness at work." The *International Journal of Mathematics and Consciousness* invites analyses of consciousness at work from various perspectives with a particular emphasis on mathematics.

2. Mathematics

Mathematics studies abstract forms, patterns, relationships, and transformations in an exact, systematic, and logical way. Forms and shapes like circles and triangles are the subject of geometry and topology. Patterns of number and operations lead to algebra. Relationships that change in time form the basis of calculus. Mathematics also includes the study of mathematics itself. The study of mathematical reasoning is undertaken by logic. Even questions about the limits of the mathematical method and the nature of mathematical knowledge can be addressed using the methodology of mathematics.

Using mathematical models of experimental observations of the physical world makes it possible to give a purely abstract formulation of real-life phenomena. Subjective mathematical reasoning, which is nevertheless entirely rigorous, applied to these models leads to new descriptions and predictions about the world.

Mathematics is fundamentally a method that finds patterns of orderliness in the subjective field of human intelligence and thought. Based on sets of axioms and postulates that are accepted without proof, mathematics gives a structure to the way our minds and intellects operate. It systematizes how individual human awareness perceives, discriminates, organizes, and expresses its own patterns of functioning. In our opinion, mathematics is certainly one of the most useful and scientifically manageable methods to study the interface between consciousness and physical phenomena.

Mathematics is in essence a subjective discipline that nevertheless allows us to organize and make sense of the physical universe in which we exist. Though subjective, it is precise and effective in objective scientific explorations. It is a fundamental and indispensable tool of all sciences, and at the same time, it is an expression of abstract human awareness and intellect.

3. Mathematics and Consciousness

The *International Journal of Mathematics and Consciousness* takes the position that methods of mathematics and mathematical modeling provide especially appropriate tools for studying the interface between consciousness and physical phenomena. As we have pointed out above, mathematics is a fundamental and indispensable tool of all sciences, and at the same time an expression of abstract human awareness and intellect. It is therefore the most precise scientifically reliable tool in the exploration of the dynamics of consciousness. It can be seen as the precise abstract representation of consciousness at work.

The ways in which human beings explore and express the experience of consciousness are as varied as nature itself. The following list contains some of the relevant sciences and other forms of human inquiry:

(1) Physics and chemistry (physical/quantum mechanical theories of consciousness at work)
(2) Biology and cognitive neuroscience (biological/electro-chemical/neural correlates of consciousness at work)
(3) Mathematics (abstract representation of consciousness at work)

(4) Psychology and cognitive sciences (objectification of subjective experiences of consciousness at work)
(5) Economics, particularly behavioral economics (production, distribution, and consumption of resources as models of the dynamics of consciousness at work)
(6) Philosophy (discursive representation of consciousness at work)
(7) Arts (subjective creative representation of consciousness at work)
(8) Religion (individual/group belief in the origins and dynamics of consciousness and consciousness at work)
(9) Spirituality (personal and totally subjective experience of consciousness at work)
(10) Study of pure consciousness itself (the field or screen "phenomenon" on which or by which all aspects of consciousness at work take place)

The *International Journal of Mathematics and Consciousness* maintains the position that of all such pursuits, mathematics, because of its rigor, depth, and effectiveness, is the most suitable discipline to study the interface between consciousness and the physical world. This *Journal* is devoted to exploring this interface using the rigorous approach of mathematics. We invite all mathematicians, scientists, and thinkers to submit papers using a mathematical approach to consciousness and "consciousness at work" in all its aspects.

Tony Nader, MD, PhD, M.A.R.R.

GEOMETRY FOR THE ARTIST: AN INTERDISCIPLINARY CONSCIOUSNESS-BASED COURSE

CATHERINE A. GORINI, PhD

Abstract. Geometry is an inherent aspect of any work of art. Both geometry and art are fundamentally products of the consciousness of individuals—the mathematicians who create geometry and the artists who create art. We therefore expect geometry and art to embody qualities of consciousness. This understanding is the basis for *Geometry for the Artist*, a mathematics course at Maharishi University of Management that explores how certain topics in geometry (symmetry, perspective, fractals, non-Euclidean geometries, and topology) are connected to art, and, moreover, how the understanding of consciousness developed by Maharishi Mahesh Yogi helps us see connections between art and geometry. This paper describes specific mathematical topics studied in this course, how they are used in art, and their relationships to consciousness.

1. Introduction

Art is subjective, depending on the emotions, intentions, life experiences, culture, training, and skill of the artist. As Kasimir Malevich (1878–1935) puts it, "Every work of art—every picture—is the reproduction, so to speak, of a subjective state of mind—the representation of a phenomenon seen through a subjective prism (the prism of the brain)" [38, p. 40]. The appreciation of art is likewise subjective, depending on subjective characteristics of the viewer.

Studio training and the study of art history can culture the ability of the artist to create art and the ability of the viewer to appreciate art. In addition, the intellectual approach of mathematics can be valuable to the artist and the viewer. According to Tony Nader:

> Certain forms of art also appeal to the intellect. The intricate symmetries and constructs in architecture and in classical music, for example, sometimes entice an intellectual analysis that helps reveal their beauty. As knowledge has organizing power, intellectual understanding of certain aspects of art awakens a greater appreciation for them. [43]

The premise of the course *Geometry for the Artist: From Point to Infinity* at Maharishi University of Management is that the study of geometry in relation to works of art promotes an intellectual understanding that can lead to greater appreciation of both geometry and art. To that end, the course covers several major topics in geometry, how artists use them, and how they are connected to consciousness. Students have found this approach valuable. They have said:

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- With a course like this, learning how unbounded consciousness is the source of all art expression and geometry has open my eyes to boundless potential for my own self-expression.
- Very well done in this course of showing the infinite value in shapes and how artist and mathematicians are very much the same in their expression. I like how we always related the lessons to Maharishi Vedic Science. I have a totally different view toward math thanks to your class.
- This class and the context it was taught in actually facilitated a connection and relationship to the knowledge.
- Consciousness gives life to knowledge and a sense of purpose to daily activities, including school and homework.

This paper will describe the course and show the value of this interdisciplinary approach.

2. Overview

To justify the premise of the course, namely the importance of geometry for the study of art, the next section, Section 3, will present evidence that artists use geometry in their work in a significant way.

Section 4 introduces Maharishi Science and Technology of Consciousness, the scientific approach to the study of consciousness used in the course, and describes its value for the students.

The next sections introduce five topics of geometry (symmetry, perspective, fractals, non-Euclidean geometry, and topology), how they appear in art, and the insights given by Maharishi Science and Technology of Consciousness.

Symmetry in art has properties of balance and harmony. The mathematical interpretation of symmetry in Section 5 demonstrates the qualities of silence and dynamism belonging to consciousness.

Perspective, discussed in Section 6, gives mathematical procedures for creating a picture that shows a scene just as the artist saw it. The description of the structure of knowledge given by Maharishi Science and Technology of Consciousness perfectly describes the situation of a perspective picture: knowledge (of the scene) is the coming together of knower (viewer), process of knowing (the picture), and the object of knowledge (the scene).

Fractals are present everywhere in nature and their representations are therefore present in art; see Section 7. Fractals are generally created by a process of repetition—or self-referral—and can appear in the work of an artist as the result of the artist's self-referral creative process.

Non-Euclidean geometries, described in Section 8, give an understanding of the properties of surfaces of natural objects. At the same time, by extending the familiar geometry of Euclid, non-Euclidean geometries show that there exists a range of possibilities for geometry, similar to the range of possibilities of consciousness.

Topology, the final area of geometry given in Section 9, is the subtlest of all the topics studied in the course. Topology extends the range of geometry from the concrete level of Euclidean geometry to a very abstract level.

Finally, we consider some themes from the study of consciousness more broadly and how they are connected to geometry and art and conclude with some students' reflections describing what they have learned from connecting geometry, art, and consciousness.

3. Do artists really use geometry?

When looking at a work of art that appears to be a beautiful and complete subjective expression of the artist, one might wonder whether the artist intentionally used the mathematical discipline of geometry in the creation of the work. One might be concerned that intellectual analysis using geometry would find something that the artist did not really intend or plan.

Our discussion below will show that artists make explicit and implicit use of geometry and will support our approach that using geometry is fruitful in the analysis of a work of art.

3.1. **Explicit use of geometry.** During the Renaissance, the influence of mathematics on artists was substantial. New appreciation for Euclidean geometry led to the modern theory and techniques of perspective, which were introduced in the early fifteenth century by Italian artists Filippo Brunelleschi (1377–1446) and Leon Battista Alberti (1404–72).

Alberti devoted a large part of his treatise *Della Pittura (On Painting)* to the application of mathematics to painting. This included a significant new development in the mathematics of perspective—the technique for constructing a checkerboard or grid in perspective—shown in Figure 1. Alberti concludes his discussion of painting with recognition of the importance of geometry for the artist, saying, “Therefore, I believe that painters should study the art of geometry” [5, p. 88].

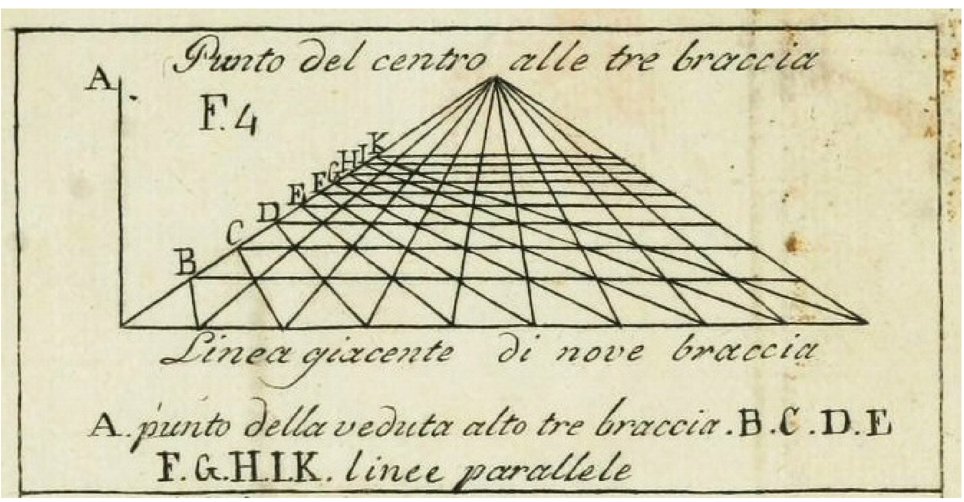

Figure 1. Diagram by Leon Battista Alberti of perspective lines leading to a vanishing point from his treatise *Della Pittura*

Other Renaissance painters who used the geometry of perspective in their work were Paulo Uccello (c. 1397–1475), noted for his dramatic and forceful use of perspective; Piero della Francesca (c. 1415–92), a mathematician and artist, who wrote the treatise on perspective *De Prospectiva Pingendi (On the Perspective of Painting)*; and Leonardo Da Vinci (1452–1519), a master of geometric techniques of perspective, as shown in Figure 2.

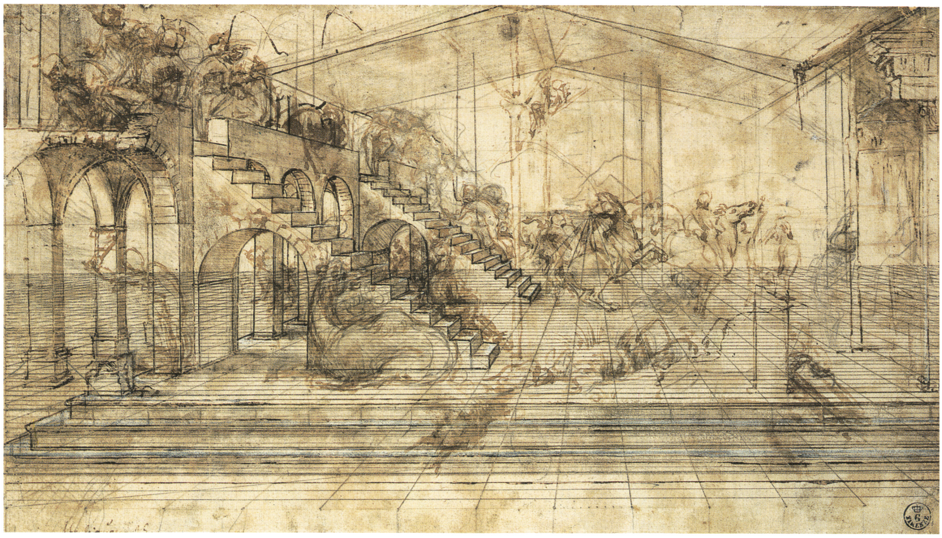

FIGURE 2. Leonardo da Vinci, Perspective study for the background of the *Adoration of the Magi*, 1481 (Dover)

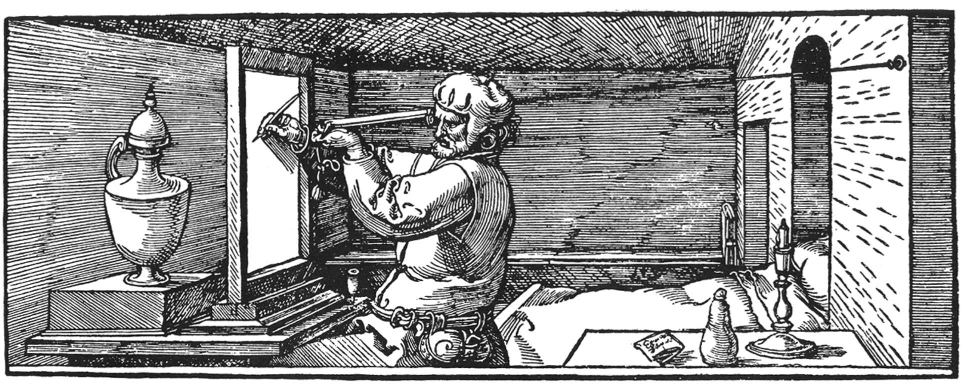

FIGURE 3. Albrecht Dürer, *A man drawing a can*, 1538 (Dover)

The German artist Albrecht Dürer (1471–1528) learned about perspective and other uses of geometry during his trips to Italy in 1495 and 1505. He later consolidated this knowledge in *Four Books on Measurement* and *Four Books on Human Proportion*, the first works in German to describe the mathematical basis of art.

The care with which he presents the theory of perspective is suggested by the woodcut shown in Figure 3. This picture from *The Art of Measurement* illustrates the concepts of station point (shown as the hook on the wall at right), visual ray (the string and tube held by the artist), and picture plane (the blank pane of glass on which the artist is drawing).

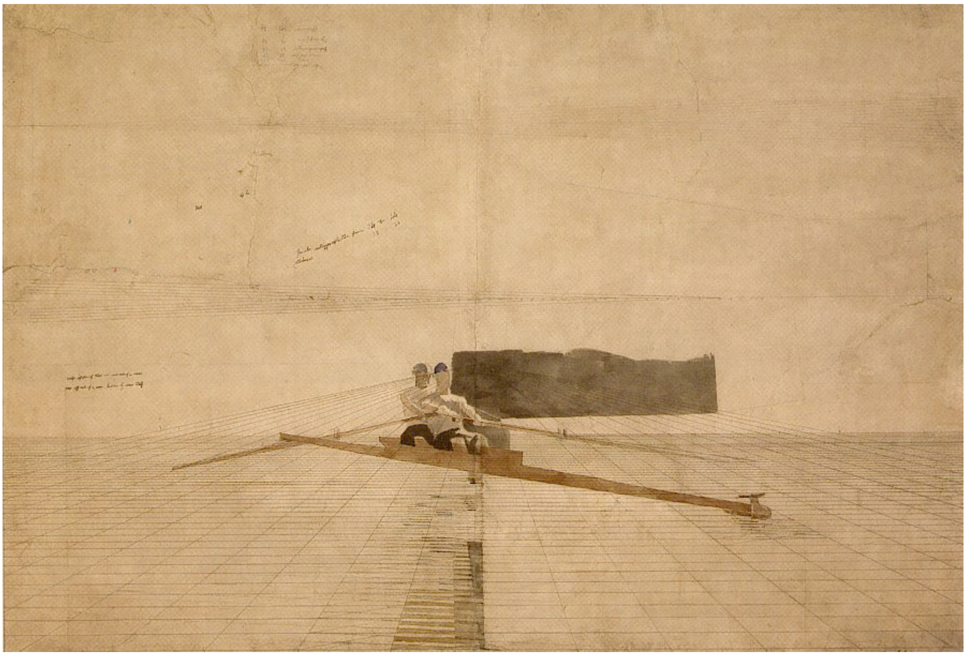

FIGURE 4. Thomas Eakins, *Perspective Drawing for "The Pair-Oared Shell,"* 1872

The American artist Thomas Eakins (1844–1916) made extensive use of perspective, creating carefully detailed perspective studies such as the one shown in Figure 4. His textbook *A Drawing Manual* [19] deals almost exclusively with perspective.

There are many examples of geometry in art other than the use of perspective. Kasimir Malevich developed a style he called *Suprematism*, which "used only geometric shapes and a limited colour range" [11]. One example is his painting *Black Square*, a black square on a white ground.

Several schools of art in the late nineteenth and early twentieth century adopted geometric themes: cubism, analytic cubism, synthetic cubism, geometric abstraction, and pointillism. Artists sometimes indicate the influence of geometry on their work in the title—*Black Circle* and *Suprematism Painting: Eight Red Rectangles* by Malevich as well as *Squares with Concentric Circles*, *Circles in a Circle*, and *Several Circles* by Wassily Kandinsky (1866–1944).

Salvador Dalí (1904–89) had a deep and lasting interest in mathematics, working closely with René Thom (1923–2002), the French topologist who introduced him to catastrophe theory [40]. Dalí's interest in topology is shown in the topologically transformed clocks of *Persistence of Memory* (1931). Thomas Banchoff (b. 1938), American geometer, helped Dalí understand higher dimensions of space [9]. The cross in Dalí's *The Crucifixion—Corpus Hypercubus* (1954) can be interpreted as the unfolding of a four-dimensional hypercube.

The unexpected and riveting work of M.C. Escher (1898–1972) was influenced by the knowledge of mathematics that he acquired from many different sources. A 1922 paper by the Hungarian mathematician George Pólya (1887–1985) [50, 51] inspired many of Escher's symmetry drawings. A picture sent to him of the Poincaré disc by geometer H.S.M. Coxeter (1907–2003) became the basis for Escher's *Circle Limit* series [51]. The Penrose tribar, an impossible figure devised by British mathematician and physicist Roger Penrose, motived pictures such as *Waterfall* and *Ascending and Descending* [21, 51].

Today, there is great sharing of ideas and techniques among artists and mathematicians. The Bridges Organization has yearly conferences [1] that bring artists and mathematicians together to "bridge" their areas of interest. The American Mathematical Society includes the yearly Mathematical Art Exhibition [2] in their annual meeting. The Mathematical Association of America has a Special Interest Group on Mathematics and the Arts.[1] Mathematics Awareness Month for 2003 [3], sponsored by the Joint Policy Board for Mathematics, was devoted to mathematics and art. *The Journal of Mathematics and the Arts*[2] was founded in 2007.

3.2. **Implicit uses of geometry in art.** We now consider how geometry appears in art in less explicit ways.

First, note that the forms and structures in nature that inspire artists are the same as those that motivate the development of geometric ideas. For example, symmetry appears over and over in nature: the bilateral symmetry of the human body; the radial symmetry of the sun; and the three-dimensional symmetry of crystals. Most flowers have symmetry, ranging from the bilateral symmetry of an orchid to the three-fold symmetry of a tulip, to the five-fold symmetry of an apple blossom, to the spiral symmetry of the sunflower. Many animals—mammals, birds, reptiles, and insects—have bilateral symmetry. Starfish and other marine life have five-fold, and sometimes seven-fold, ten-fold, or even fifty-fold, symmetry [28].

Artists inspired by such symmetrical structures in nature will of course incorporate symmetry into their work. We see this, for example, in full-face portraits and masks and sometimes in flower or animal paintings. Naturally occurring symmetrical designs are often the inspiration for symmetrical designs, borders, and tilings, as seen in the medieval border in Figure 5 that is based on a flower motif.

The mathematical study of symmetric patterns was motivated in part by the need of crystallographers to describe the symmetry of crystals [52, p. 16ff].

Geometric laws govern how light travels and how we see the shapes around us. The laws of perspective, based on these geometric laws, determine how three-dimensional shapes appear to the eye. Thus, an artist obeys the geometry of perspective when painting what is seen onto a two-dimensional canvas, even though not explicitly using mathematical rules. For rectilinear shapes in the man-made

[1] `http://sigmaa.maa.org/arts/`

[2] `http://www.tandfonline.com/toc/tmaa20/current`

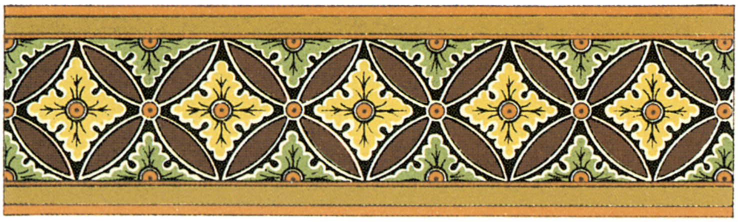

FIGURE 5. A Medieval Border, France

environment, of course, these geometric laws are very concrete and striking, but these laws also apply when looking at and painting a natural scene.

As Maharishi points out [24, p. 41], "Art is the expression of life; it is the expression of creation." Geometry is found everywhere in nature, so when an artist draws whatever shapes are found in nature, the images will naturally have the geometry belonging to those natural objects—the symmetry of flowers and crystals, the fractal geometry of trees and mountains, and the non-Euclidean geometry of the human body.

3.3. **Using geometry to analyze art.** We have seen that artists use many types of geometry in their work and paint a variety of natural geometric forms. Thus, it is necessary to use a range of geometric concepts in the analysis of specific works of art. For example, symmetry transformations are convenient for the mathematical analysis of designs, borders, and tilings. Symmetry classifications allow us to compare the symmetry of different patterns.

An understanding of perspective is necessary for analyzing a picture that was created using perspective. Geometrical methods can determine the station point (the location of the artist or viewer) and, in many cases, the viewing distance (the distance of the artist or viewer from the canvas).

Identifying congruent or similar shapes in a painting gives insight into what shapes the artist intended to connect or relate to one another. Locating the presence of fractals, curved lines and surfaces, or distortions through topological transformations gives an appreciation for how the artist viewed, interpreted, and expressed the subject matter.

Analysis of pictorial composition requires looking at a picture in terms of lines and shapes, the arrangement of shapes, and the balance created by those shapes. Such analysis deals mainly with the overall geometric structure inherent in the work [49]. For example, the dominant shapes in a painting may be arranged in a triangular or circular shape. The artist may use lines to emphasize different qualities: horizontal lines like the horizon between sky and sea give a feeling of expansion; vertical lines like trees or columns give stability; and diagonal lines like a plane taking off convey dynamism.

While the geometrical analysis of a work of art does not completely capture the full value of a work of art, it can give unique insight not available in any other way. Geometrical analysis can support and confirm conclusions from other types of analysis, and it gives a richer appreciation of the skill of the artist.

With this appreciation of the usefulness of geometry for the artist, we now turn to discussing various aspects of the Consciousness-Based course *Geometry for the Artist*.

4. The Value of Maharishi Science and Technology of Consciousness for the Student

Along with gaining objective knowledge of a discipline such as art or geometry, the student should be developing subjective qualities such as intelligence, focus, and creativity. To accomplish this, Maharishi University of Management integrates Maharishi Science and Technology of Consciousness into the structure of each academic course.

Maharishi Science and Technology of Consciousness, like any science, has two components, practical or experiential and theoretical [37, p. 271]. The Transcendental Meditation and TM-Sidhi program provides the experiential element of this science of consciousness. The Transcendental Meditation technique is a simple, natural technique practiced for twenty minutes twice daily. During the practice of the Transcendental Meditation technique, the mind settles down to its least active state, and the meditator gains the subjective experience of wakefulness without active thinking, a state of silence without activity, a state of pure awareness or pure consciousness. The TM-Sidhi program cultivates the mind to think and act from that least excited state of awareness.

The theoretical component of this science, developed by Maharishi Mahesh Yogi over a fifty-year period, is based on three sources: the wisdom contained in the ancient Veda and Vedic literature (see for example [44, pp. 1, 35 ff.]), intellectual analysis of personal experiences of higher states consciousness [46], and scientific research on the development of higher states of consciousness [4, 13, 14, 15, 45].

Maharishi Science and Technology of Consciousness is incorporated into the curriculum at Maharishi University of Management in two ways. First, students practice the Transcendental Meditation and TM-Sidhi program as part of their academic program. Second, students gain a holistic understanding of all disciplinary content by connecting the principles of each discipline they study with the principles of personal development that Maharishi has advanced; see for example [18, 33, 35].

Students gain many benefits from their regular meditation. During the practice of the Transcendental Meditation technique, the active mind, which is ordinarily processing thoughts and sensory impressions, becomes progressively less active until it transcends, or goes beyond, thoughts and sensory input and experiences its own nature, pure unbounded awareness. With regular practice, students become familiar with their own consciousness, the subtlest level of life. This leads to refinement and expansion of the awareness of the meditator outside of meditation, so that students gain a deep, personal connection with the qualities of consciousness that artists draw upon to structure a work of art. They become viewers capable of experiencing the full value of a work of art, from the level of the colors and shapes on the canvas to the finest value of emotion that the artist has embedded in those colors and shapes [24, p. 33].

Scientific research [4, 13, 14, 15, 45] has documented the enhancement of many characteristics important for students and relevant to the appreciation of art. Intelligence grows with the regular practice of the Transcendental Meditation technique [6, 17, 54] as does creativity [30]. Improved efficiency of visual perception and increased freedom from habitual patterns of perception and increased perceptual flexibility [16] give students a fresh approach when looking at a work of art. Greater aesthetic orientation [47] leads to greater appreciation of art. Research also shows that college art students develop broader comprehension and improved ability to focus attention as well as greater field independence with regular practice of the Transcendental Meditation program [25, 26, 57].

5. Symmetry

All cultures use symmetric designs for their symbols, flags, and crests. Artisans everywhere decorate pottery, fabric, and buildings with symmetric designs. Symmetry is associated with qualities of balance, harmony, orderliness, and coherence. It should not be surprising that symmetry is also a quality of the field of pure consciousness, as we shall see below.

5.1. **Symmetry transformations and their classifications.** Mathematicians use symmetry transformations to measure the degree of symmetry belonging to a mathematical structure or physical object.

A *symmetry transformation* is a motion of a shape or design that leaves the shape or design apparently unchanged. To illustrate this, consider the designs shown in Figure 6. The first design would look the same if it were flipped across a vertical line through its center as would the second. Both of these designs are said to have *bilateral symmetry*, a very familiar type of symmetry. The third and fourth designs would also look the same if they were flipped across vertical lines through their centers, but in addition would look the same flipped across horizontal and diagonal lines through their centers. These designs have *four-fold reflection symmetry.* In addition, both of these designs have four-fold rotational symmetry, which means they look the same when rotated through angles of 90°, 180°, 270°, or 360°.

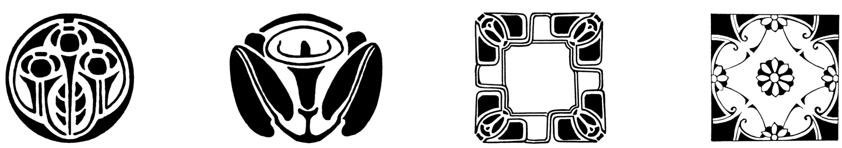

Figure 6. Four symmetric designs. The two on the left have bilateral symmetry and the two on the right have four-fold reflection symmetry.

Moving a shape in such a way that it looks the same captures the intuitive idea of symmetry. If we reflect a shape across a line and it looks the same, we have shown that the two halves of the shape on opposite sides of the line look identical. Similarly, if we rotate a shape 90° about its center point and it looks the same, we have captured how four different parts of the shape look identical.

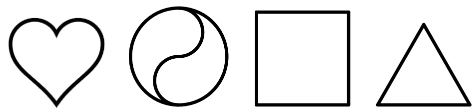

FIGURE 7. Four simple symmetric geometric designs

To determine the symmetry transformations belonging to a shape, we imagine that it is being moved. The first shape in Figure 7, the heart, is symmetric because it would look the same if it were reflected across a vertical line through its middle; each half is a mirror image of the other half, as shown in Figure 8. The second shape, the yin-yang symbol, is symmetric because it would look the same if it were rotated $180°$ about its center point. The square and the triangle have both mirror and rotational symmetry. The square would look the same if it were reflected across vertical, horizontal, or diagonal lines or if it were rotated through angles of $90°$, $180°$, $270°$, or $360°$. The triangle would look the same if reflected across vertical or diagonal lines or rotated through angles of $120°$, $240°$, or $360°$. The dashed lines in Figure 8 show the mirror lines of the three shapes.

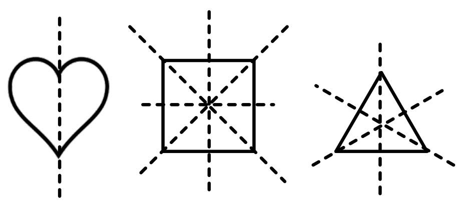

FIGURE 8. Three symmetric designs along with their mirror lines

The connection of the mathematical characterization of symmetry with consciousness is illustrated by verse 18 of Chapter 4 of the Bhagavad-Gita, where Krishna explains the relationship of action and inaction. The translation of this verse by Maharishi Mahesh Yogi [37, p. 278] is:

He who in action sees inaction and in inaction sees action is wise among men. He is united, he has accomplished all action.

From the perspective of this verse, a symmetry transformation, such as a reflection or rotation, is an action imposed on an otherwise inactive mathematical object, so the transformation "sees action in inaction." An essential characteristic of a symmetry transformation is that the object looks the same after the symmetry transformation as it did before—the viewer cannot tell the difference between the object before the transformation and the object after the transformation. Thus, a symmetry transformation "sees inaction" of the design "in action," under the movement of the symmetry transformation.

By inaction, Maharishi means the state of Being, Transcendental Consciousness, that state of pure awareness that can be experienced through the practice of the Transcendental Meditation technique. One who sees "inaction in action" is one who experiences this state of pure awareness along with the ordinary activity of daily life. One who sees "action in inaction" experiences all activity in terms of the silence or inaction of the deepest, non-active level of the Self, the state of Being. Such an individual is fully realized and has gained the highest level of consciousness.

An individual who experiences silence along with activity and sees all activity as an expression of silence is, in Maharishi's translation, "united, he has accomplished all action." This means such an individual has attained perfection and gained fulfillment [37, p. 280]. Action is a way of fulfilling one's desire. To have "accomplished all action" means to have attained all possible goals in life, indicating that one has gained fulfillment. For the mathematician, gaining complete knowledge of the symmetry of some mathematical structure is very powerful and fulfilling.

5.2. **The beauty of symmetry.** Something that has symmetry exhibits qualities of silence or inaction along with qualities of dynamism or action. A basic shape that is not changed—silence—is repeated over and over in different positions or orientations—dynamism.

Symmetry is beautiful and fascinating; it is found everywhere in nature; and it is a prevalent theme in art, architecture, and design in cultures all over the world and throughout human history. From the charm of a snowflake to the deep spirituality of Leonardo's *Last Supper*, symmetry has an essential role in nature and art.

Pure consciousness has remarkable qualities of symmetry. Pure consciousness is unbounded and everywhere the same. Every "part" of pure consciousness looks like every other "part." Any movement of pure consciousness leaves it unchanged, so every movement of pure consciousness is a symmetry transformation. Thus, the collection of symmetry transformations of pure consciousness includes all transformations. For this reason, it makes sense to say that pure consciousness has the greatest possible symmetry.

This analysis concurs with Maharishi's description of the importance of maintaining symmetry in physical systems. He points out the quality of symmetry that belongs to consciousness [34, pp. 181–2]:

> Maintenance of symmetry also applies to consciousness: pure consciousness, self-referral consciousness, unbounded awareness, is the most expanded, smoothest state, the one with the most expanded boundaries—it has the greatest degree of symmetry.

To see why symmetry is so attractive and aesthetically pleasing to us, in art as well as in science, consider the field of pure consciousness. According to Maharishi [37, p. 282], the silent level of life, pure consciousness, the source of thought, is subjectively experienced as bliss; whenever the active level of the mind begins to move in the direction of the silent level of the mind, it experiences increasing bliss.

The repetition of parts of a symmetrical design indicates an underlying pattern or unifying value for something that is physical and concrete. When observing a symmetric object, whether an artistic design or a mathematical structure, the mind is spontaneously led to experience the surface value of the object (activity) and the more unifying symmetric values of the object (silence) simultaneously. The charm of symmetry for the viewer is a result of this evolutionary experience of perceiving the diversity of the surface level and the unity of the silent level simultaneously.

For the scientist, the symmetry of a system is a very deep level of the organizing power that structures an object or physical system. Indeed, the most important laws of physics are those that encode the symmetry of a system [53, 58].

5.3. **M.C. Escher and Symmetry.** M.C. Escher is an artist who has used symmetry effectively. He began his intensive work with symmetry after visiting the Alhambra in Spain, where the walls, ceilings, and floors were covered with symmetric tiling patterns. Escher's purpose in creating symmetric patterns was for capturing something deeper and more powerful, for "expressing unboundedness in an enclosed plane that is bound by specific dimensions, while retaining the characteristic and fascinating rhythm" [22, p. 84]. The wide appeal of Escher's symmetric work confirms his success in this undertaking.

6. Perspective

The goal of perspective is to represent a three-dimensional scene on a two-dimensional canvas that gives the viewer the impression of viewing the three-dimensional scene rather than the two-dimensional painting. Geometric techniques of perspective make it easy and straightforward to make rectilinear shapes such as buildings, roads, fences, furniture, and tilings on a canvas appear three-dimensional.

Geometric analysis of the perspective in a picture tells us where the eye of the artist was located with respect to the canvas when the picture was painted, which is where the viewer should be located to see the picture as the artist intended. Knowing the location of the viewer—whether looking from above, below, near, far, straight on, or at an angle—indicates how the artist is connecting the viewer to the scene. The viewer might feel to be an intimate part of the activity in the picture, overwhelmed by the drama or significance of the events, or as if a dispassionate bystander. In many medieval religious works, for example, the viewer is uninvolved. Renaissance paintings make viewers feel as though they are intimate and involved. Impressionist paintings, on the other hand, don't give a clear-cut location for the viewers but take the viewer into the mind and heart of the artist.

The woodcut *An artist drawing a seated man on to a pane of glass through a sight-vane* by Albrecht Dürer, shown in Figure 9, is an example of a picture that makes us feel as though we are an intimate part of the activity. This picture is drawn using perspective while also showing the elements of constructing a perspective picture: the station point where the eye of the artist is located, the picture plane on which he is drawing, and the scene he is painting. Analysis of the perspective tells us that the viewer of this woodcut is in the room, close to the scene, at eye level with the standing artist, between the artist and the seated man, observing the activity of the artist in a familiar, personal way.

Many artists do not adhere rigidly to the rules of perspective, manipulating them to achieve a specific result. For example, in *The Resurrection*,[3] Piero della Francesca lets the viewer look directly into the face of Christ even though the viewer is situated below the soldiers guarding Christ's tomb at the base of the picture. Pablo Picasso (1881–1973), in paintings such as *Girl before a Mirror*,[4] shows us the subject from

[3] https://commons.wikimedia.org/wiki/File:Resurrection_(Piero_della_Francesca).jpeg
[4] https://www.moma.org/collection/works/78311

FIGURE 9. Albrecht Dürer, *An artist drawing a seated man on to a pane of glass through a sight-vane*, 1525, woodcut (Dover)

more than one point of view. M.C. Escher bends the rules of perspective just enough to deceive the viewer, as described below in Section 6.2. Applying the rules of perspective helps the viewer understand the intention of the artist in pictures such as these.

6.1. **Knower, known, and process of knowing.** Maharishi Science and Technology of Consciousness helps us understand the relationship of artist, viewer, painting, and scene, which are so significant in perspective pictures.

Looking at a picture is a process of gaining knowledge. Maharishi maintains that any experience of knowledge has three components:

> Knowledge naturally involves three things: the knower, the object of knowledge, and the process that connects the knower and the object—the process of knowing. [24, p. 90]

The value of a work of art depends on the quality of all three components. Of these three, Maharishi identifies the component of the knower as key:

> However, without having established the "I"—the subjective aspect of knowledge—the object will not be fully located. In the field of knowledge, it is necessary that the knower be established first, and on the basis of establishing the knower, the object is known. From this analysis, we can see that very naturally the knower is the first point of reference in knowledge, the object of knowing is the second point of reference, and the process of knowing that connects the two is the third point of reference. [24, pp. 90–91]

For this reason, the development of the subjective aspect of the knower—the viewer—is essential for full appreciation of any work of art. This is particularly true for a perspective painting since the viewer has such an essential role. And, as we have seen in Section 4, students' regular practice of the Transcendental Meditation technique enhances the development of inner subjective qualities and perceptual abilities.

6.2. **M.C. Escher and the manipulation of perspective.** Escher's woodcuts from his earlier years in Italy demonstrate that he was a master of perspective, carefully positioning the viewer with respect to a three-dimensional scene to create a specific effect. In the woodcut *Tower of Babel*[5] from 1928, Escher used three-point perspective to locate the viewer high above the construction activity of the tower, giving a feeling of the height and grandeur of the tower. About *Cubic Space Division*[6] and *Depth*,[7] Escher said, "My only intention was to suggest an impression of three-dimensionality, of endless depth" [22, p. 56] and he succeeded admirably in this.

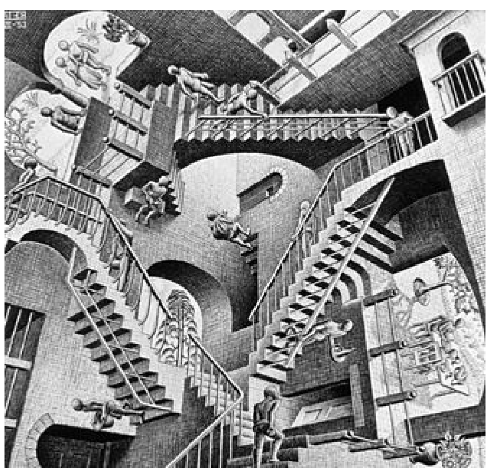

FIGURE 10. M.C. Escher, *Relativity*, 1953, lithograph

[5] `http://www.mcescher.com/gallery/italian-period/tower-of-babel/`
[6] `https://www.nga.gov/Collection/art-object-page.54254.html`
[7] `http://www.mcescher.com/gallery/recognition-success/depth/`

FIGURE 11. M.C. Escher, *Waterfall*, 1961, lithograph

In his later works, Escher carefully used variations of linear perspective to create images representing impossible realities. To understand Escher's intention and to gain full appreciation of his skill, the viewer must understand the mathematical properties of perspective. The lithograph *Relativity*, which uses perspective correctly, shows sixteen people walking, sitting, and climbing stairs; see Figure 10. When each person or group is viewed individually, everything looks fine. But when the picture is looked at as a whole, we see that the alignment changes from group to group. The zenith of one group is the nadir of a second group, the right-hand vanishing point of a third group, and the left-hand vanishing point of yet a fourth group. These inconsistencies completely confuse the viewer, as Escher intended.

In pictures like *Waterfall*, Figure 11, Escher uses the principle that an object farther away from the picture plane appears higher in the picture. The viewer should interpret the zigzag of water that travels higher in the picture as water that is moving farther away; however, Escher connects the highest point of this zigzag of water to the top of the waterfall, which appears to be close. This makes the viewer interpret the water to be going up rather than away and Escher's mastery of perspective again confuses the viewer.

7. Fractals

Benoit Mandelbrot (1924–2010), one of the founders of fractal geometry, asserted that the geometry of Euclid was not the most effective way to study shapes seen in nature, but rather it was fractal geometry that could best model nature. He referred to this geometry as the "geometry of nature" [39, p. 1] because natural shapes such as clouds, mountains, waves, coastlines, rivers, ferns, and trees are fractals. The fractal shape of lightning is shown in Figure 12.

FIGURE 12. Lightning.

To describe the structure of fractals, we need the concept of similarity: two shapes are *similar* if they have the same shape but possibly different sizes, as shown in Figure 13. Scaling one of the two similar shapes up or down can make it look exactly like the other.

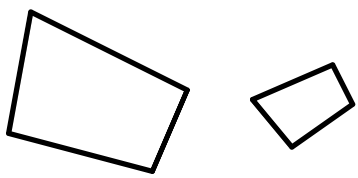

FIGURE 13. Two similar shapes

A fractal is a geometric shape that is similar to itself; this is the property of *self-similarity.* This means that a fractal looks like itself when scaled up or down. This is shown in the Koch snowflake curve, Figure 14, where the top part of the curve is similar to the whole curve; the top part looks like the whole curve when scaled up.

7.1. **The self-referral structure of fractals.** The self-referral construction of a fractal is an example of the self-referral dynamics of consciousness described by Maharishi [35, pp. 10–11]. The silent value of pure consciousness interacting with itself is the fundamental self-referral process of creation [35, p. 185]. Maharishi sees the world around us as the result of this self-referral dynamics of the field of intelligence, which is the same unified field of natural law recognized by physicists:

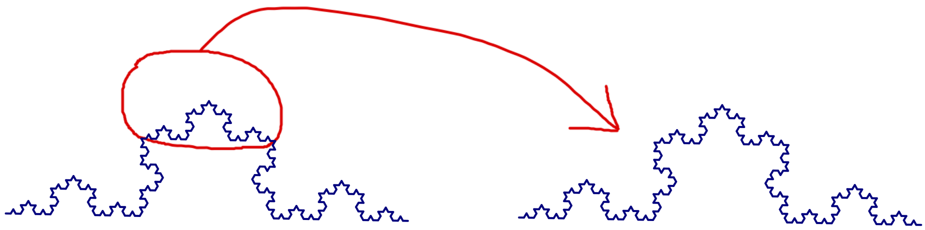

Figure 14. When a small part of the Koch snowflake curve is scaled up, it looks like the whole curve.

> Both understandings, modern and ancient, locate the unified source of Nature's perfect order in a single, self-interacting field of intelligence at the foundation of all the Laws of Nature. This field sequentially creates, from within itself, all the diverse Laws of Nature governing life at every level of the manifest universe. [34, p. 78]

This is like the mathematical process of creating a fractal through an iterative process as seen in the next section.

7.2. **Construction of Fractals.** Fractals illustrate how the self-referral dynamics of a simple system can create extraordinary diversity, parallel to the way that pure consciousness, interacting with itself alone, gives rise to the full range of creation as described above.

Creating a fractal is really nothing more than making scaled copies of an original basic shape over and over. The basic shape is like the silent "self" in the process of self-referral and the repetition and scaling are like the dynamic value of creation.

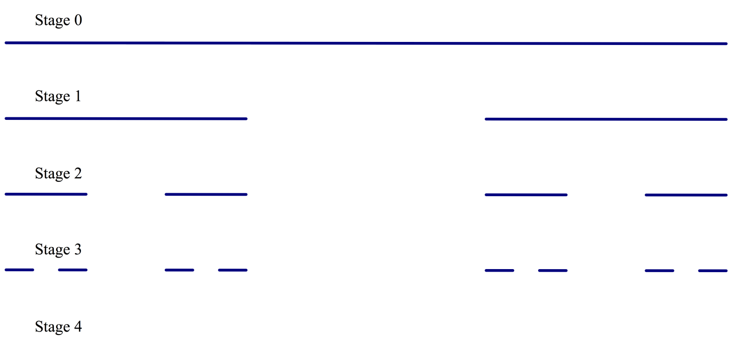

Figure 15. First stages of the Cantor set

To construct the Cantor set, for example, begin with a straight line segment (Stage 0); see Figure 15. Remove the middle third of the segment, leaving the two congruent segments of Stage 1. Each of these two segments is similar to the original segment, scaled down by a factor of 1/3. To obtain Stage 2, remove the middle segment of each of these two segments. Note that in Stage 2, we have four similar copies of Stage 0, but two similar copies of Stage 1. Continue on in this way, removing the middle third of each segment belonging to one stage to get the smaller segments belonging to the next stage. The Cantor set is the set of points that remains after this process has been performed infinitely many times; at the final stage, the Cantor set is similar to a half of itself, a quarter of itself, or an eighth of itself—and so on infinitely.

Georg Cantor (1845–1918), the originator of the Cantor set, discovered many of its surprising properties: it has infinitely many points; it has just as many points as the original segment; it consists only of individual points; it does not contain any segments at all, even though at each successive stage there are more and more segments.

FIGURE 16. Edge and its replacement in the iterative construction of the Koch snowflake

Another simple fractal is the Koch snowflake. Stage 0 is an equilateral triangle. To get Stage 1, replace the middle third of each edge of the triangle with two sides of an equilateral triangle, as shown in Figure 16. Stage 1 has 12 edges, as shown in Figure 17. At Stage 2, there are 48 edges, and so on. Keep repeating this procedure, replacing each edge with the four edges of Figure 16, scaled to fit the edge that it replaces. The Koch snowflake, Figure 18, is the shape that results as the end stage of infinitely many iterations of this procedure. Like the Cantor set, it is a purely abstract mathematical structure that cannot be drawn on paper.

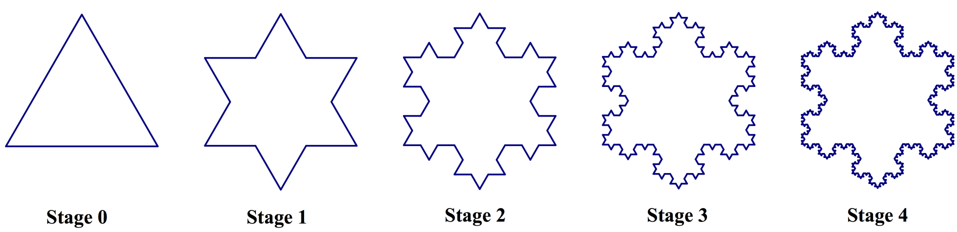

FIGURE 17. First stages of the Koch snowflake

The construction of these fractals illustrates very clearly the description of the self-referral structure of consciousness given by Maharishi in verse 8 of Chapter 9 of the *Bhagavad-Gita*:

Prakṛitiṁ swām avashtabhya visṛijāmi punaḥ punaḥ
Curving back upon My own Nature, I create again and again.
[35, p. 37]

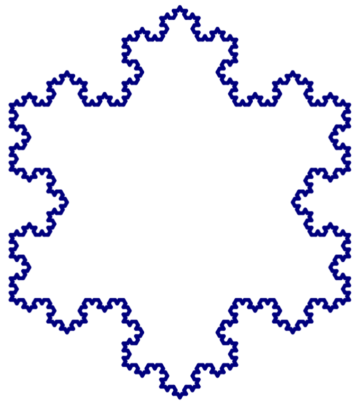

Figure 18. The Koch snowflake

7.3. **Fractals in art.** Artists recognized and imitated the fractal structure of their natural environment even before mathematicians began to study fractals. For example, fractal structures appear in many African designs [20], Celtic illuminated manuscripts [41], and Persian carpets [42]. *The Great Wave off Kanagawa* by Japanese artist Hokusai (1760–1849) shows the fractal structure of an ocean wave. Modern architecture also uses fractal-like structures [56, pp. 325–354, 513–524]. Fractal geometry in art may also be a reflection of the way that artists create their work; Bales [7, 8] proposes that the fractal appearance of certain quilts is the result of the iterative way the quilters work.

8. Non-Euclidean geometry

For thousands of years, there was only one known geometry—the geometry common to the ancient Hindus, the ancient Egyptians, the ancient Greeks, the Mayans, and others [12, 31]. Euclid (c. 330–c. 270 BCE) formalized this geometry in *The Elements* [23], a systematic development of geometry from first principles. Long regarded as the model of presenting knowledge, *The Elements* lists postulates and common notions, the assumptions or rules that guide the development of Euclidean geometry. From those first fundamental assumptions, Euclid derived all of his propositions logically and sequentially. In this way, Euclidean geometry is structured in layers, from the subtlest foundational layer of postulates and common notions through more expressed layers of elementary propositions, to the complex layers that include the Pythagorean theorem and the construction of the five Platonic solids.

With this firm foundation, Euclidean geometry was considered to be the only possible geometry until several major discoveries were made in the nineteenth century, when János Bolyai (1802–60) and Nicolai Lobachevsky (1792–1856), working separately, discovered another kind of geometry [55]. They questioned one of Euclid's postulates, the fifth or parallel postulate, which says that given a line ℓ and a point P not on the line, there exists one and only one line ℓ' through the point

P that is parallel to the original line ℓ, as shown in Figure 19. By changing the parallel postulate—in other words, by operating on the subtlest layer of Euclidean geometry—they were able to create a new geometry, called hyperbolic geometry.

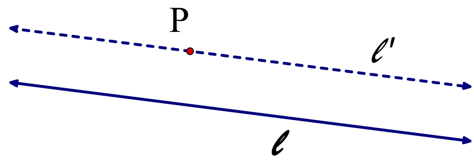

FIGURE 19. The unique line ℓ' through point P that is parallel to the line ℓ .

This new geometry was recognized to be just as valid, just as consistent, and just as true as Euclidean geometry only when, in 1868, Eugenio Beltrami (1835–1900) created a model for hyperbolic geometry, the pseudosphere, from within the structure of Euclidean geometry.

Hyperbolic geometry was joined by another new geometry, elliptic geometry, developed by Bernhard Riemann (1826–66). He showed how the surface of the sphere also satisfied all of Euclid's postulates except the parallel postulate. These new geometries were shown to be of practical as well as theoretical interest when Albert Einstein (1879–1955) used them in his theory of general relativity [48].

Furthermore, Bernard Riemann's study of the sphere showed that elliptic geometry is just as consistent, valid, and true as the other two geometries. Geometry was now not one possibility but a field of possibilities.

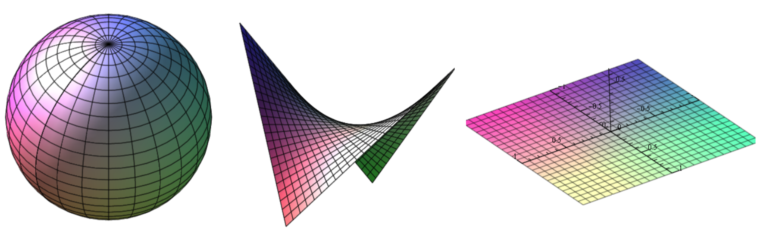

FIGURE 20. Geometries. The surface on the left has Riemannian or elliptic geometry, the surface in the middle is hyperbolic, and the Euclidean plane is on the right.

These geometries look different, as shown in Figure 20. We see the shapes associated with these geometries everywhere around us. The flat surfaces of buildings and man-made objects are examples of Euclidean geometry. Round or spherical objects, like oranges, apples, a tennis ball, and the human head, are examples of elliptic geometry. The surfaces of hyperbolic geometry are like saddles or ruffles and can be seen in the inner curve of the elbow or in kale leaves.

Art students will find these three geometries and their interpretations everywhere in art. The flat surfaces of architecture and man-made items belong to Euclidean geometry. Natural forms like the human body, fruit, flower petals, and so on belong to non-Euclidean geometry. It is not uncommon to see an artist create a striking contrast between the Euclidean geometry of architecture and other man-made forms on the one hand and the non-Euclidean geometry of living forms on the other. *My Parents*[8] by David Hockney (b. 1937) is an example of this.

Artists may depict the surfaces they see realistically, as when Leonardo da Vinci drapes a flat Euclidean cloth over a round elliptic knee, shown in Figure 21. Other artists distort the geometries they see. Pablo Picasso (1881–1973) and the cubists flatten curved surfaces until they become Euclidean; see Picasso's *Portrait of Ambroise Vollard*[9] for an example. Other artists, like Diego Rivera (1886–1957) in *The Flower Carrier*,[10] emphasize the spherical nature of what they see. Kazimir Malevich,[11] Piet Mondrian (1872–1944), and the Minimalists were fascinated by flat surfaces. M.C. Escher based his *Circle Limit*[12] series on the Poincaré disc model of the hyperbolic plane [22, pp. 125–126].

FIGURE 21. Leonardo da Vinci, Study of a Drapery for the Virgin in the *Virgin and Child with St. Anne and a Lamb*, 1503 (Dover)

Through analysis of the types of geometry in a work of art, the viewer is able to see the diversity of life presented by the artist, to see how the artist integrated the diversity into the wholeness of the work, and to more deeply appreciate the skill and intentions of the artist.

[8] http://www.tate.org.uk/art/artworks/hockney-my-parents-t03255

[9] http://www.arts-museum.ru/data/fonds/europe_and_america/j/1001_2000/7199_Portret_Ambruaza_Vollara/index.php?lang=en

[10] https://www.sfmoma.org/artwork/35.4516

[11] See *Untitled*, https://www.guggenheim.org/artwork/2601, for example.

[12] http://www.mcescher.com/gallery/recognition-success/circle-limit-iii/

9. Topology

The subtlest kind of geometry is topology, which uses only the very simple relationships of set theory—membership and inclusion—for its definition. In fact, a topological space is simply a set of points together with certain relationships on the subsets of that set. This level is prior to measurement of length, area, and angle, so topology concerns itself only with properties of the organization of points independent of the measurement of length, area, or angle. In topology, two different shapes are considered to be equivalent or indistinguishable if one can be stretched or twisted into the other without tearing, cutting, or gluing. We could imagine that shapes are made of very stretchy rubber that can be expanded or contracted at will. For this reason, topology is frequently called "rubber sheet geometry." Thus, a square, circle, and triangle are topologically equivalent because any one of the three can be stretched into each of the others.

A famous example used by topologists is that a "donut" and "coffee cup" are topologically equivalent; Figure 22 indicates how a donut-shaped topological space could be transformed without cutting or gluing into a cup-shaped space. An interesting example of a topological space is the Möbius strip, which can be made from a strip of paper that has been twisted by one half-turn and joined, as shown in Figure 23.

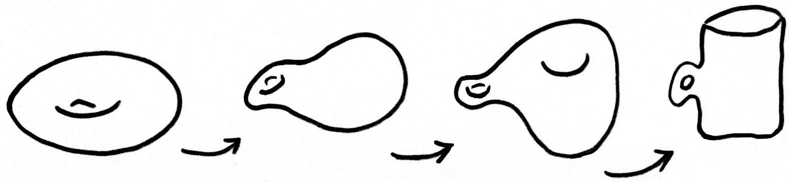

Figure 22. A topological transformation of a donut into a coffee cup

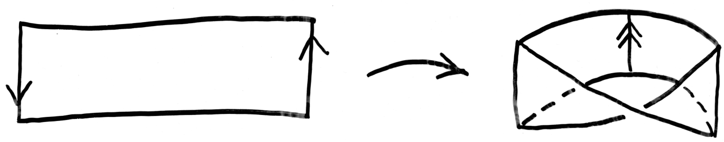

Figure 23. Construction of a Möbius band from a strip of paper. The upward-pointing and downward-pointing arrows are matched after rotating an end of the paper through one-half twist.

Many artists are fascinated by the topological transformations that stretch, twist, and bend familiar shapes. El Greco (1541–1614) and Amedeo Modigliani (1884–1920) both elongated the human figure. Salvador Dali (1904–89) stretched and warped objects in paintings such as *The Persistence of Memory* and *The Elephants*. In his *Bathers* paintings, Picasso stretches and bends his subjects almost beyond recognition. Sculptor Alberto Giacometti (1901–66) stretched the human form until it was almost thread-like; Constantin Brâncuşi (1876–1957) created the sculptures *Kiss* and *Bird in Space*, which are quite radical topological transformations of their declared subject matter. Other artists such as M.C. Escher and Max Bill (1908–94), were fascinated by the specific surfaces studied by topologists. Max Bill made versions of the Möbius band in granite, bronze, and concrete. Robert R. Wilson (1914–2000) designed a stainless steel sculpture in the shape of a Möbius band for the FermiLab in Illinois.[13] Keizo Ushio (b. 1951) is a Japanese stone sculptor who effectively uses Möbius bands in conjunction with other shapes [27].

M.C. Escher gives us a few unforeseen lessons in topology. The woodcut *Möbius Strip II (Red Ants)* in Figure 24 has ants crawling on the full length of a lattice-work Möbius strip, showing that a Möbius strip has only one side. *Möbius Strip I* in Figure 25 shows that cutting a Möbius strip down the middle leaves it in one piece.

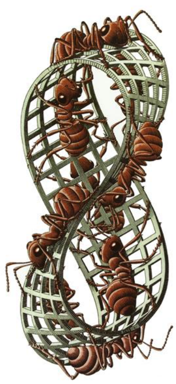

FIGURE 24. M.C. Escher, *Möbius Strip II (Red Ants)*, 1963, woodcut

FIGURE 25. M.C. Escher, *Möbius Strip 1*, 1961, wood engraving and woodcut

Life is structured in layers. Maharishi explains that the subtlest layers of life are the most powerful. Pure consciousness is the subtlest layer of life and is therefore the most powerful [36, pp. 4–5]. In this section, we have seen that topology is a very subtle layer of geometry, and it should follow that topology is a very powerful branch of mathematics. In fact, topology has applications in some of the subtlest and most powerful areas of science and technology: M-theory in physics, data analysis, quantum computing, and the study of DNA and neural networks in biology.

[13]`https://history.fnal.gov/sculpture.html`

10. Maharishi Science and Technology of Consciousness

In the previous five sections, we have discussed how specific topics in geometry are connected to art and we have seen these connections related to Maharishi Science and Technology of Consciousness. In this section, we will consider several topics in the science of consciousness more broadly and see their connections to geometry and art. For students in the course *Geometry for the Artist*, these themes support their understanding of geometry and the analysis of specific works of art.

10.1. **Consciousness is infinite and unbounded.** The field of pure consciousness is experienced during the practice of the Transcendental Meditation technique to be infinite and unbounded [44, p. 13]. Maharishi describes pure consciousness in this way:

> It is the unlimited vastness of pure existence or pure consciousness, the essential constituent and content of life. It is the field of unlimited, unbounded, eternal life, pure intelligence, pure existence, the Absolute. [36, p. 8]

This is especially relevant to the student of art and mathematics; Maharishi states that the goal of art and the goal of science are both infinity [24, p. 39]. Further, he uses the measure of unboundedness in a work of art as an indication of its value:

> Art is a way of expression that can belong to all of the five senses, as well as to the mind, the intellect, and the ego; art is a way of expression. If that expression indicates the direction of unboundedness, immortality, and bliss, if it inspires those values and indicates those qualities of pure consciousness, then it is to be considered successful art. Through the means of one sense, it takes the viewer to unboundedness, which eventually he sees as his own Self. So the unfoldment of the Self in greater degrees is the purpose of art. [24, pp. 291–292]

Students experience the field of pure consciousness in their meditation and can use this experience of unboundedness as a tool to help them go more deeply into mathematics and art.

The presence of infinity and unboundedness is everywhere in geometry. The Euclidean plane and the lines it contains extend infinitely without boundary. Even a line segment of finite length has infinitely many points. Fractals are defined in terms of an infinite sequence of iterations of a geometric construction. Topology studies the infinite variety of all possible topological spaces.

Many artists have endeavored to give a feeling or sense of the infinite in their work. Students with regular experiences of the infinite are able to more easily resonate with this feeling of infinity in a work of art.

Max Beckmann (1884–1950) made it very clear that his work expresses the infinite, invisible field that lies beyond the finite visible world around us:

> What I want to show in my work is the idea which hides itself behind so-called reality. I am seeking for the bridge which leads from the visible to the invisible, like the famous cabalist who once said: "If

> you wish to get hold of the invisible you must penetrate as deeply as possible into the visible." [29, p. 167]

M.C. Escher in his essay "Approaches to Infinity" brings out the artist's desire to portray the infinite in art, "to penetrate all the way into the deepest infinity right on the plane of a simple piece of drawing paper by means of immovable and visually observable images" [22, p. 123]. He describes his own attempts to represent the infinite using geometric principles and finds that "[t]here is something breathtaking in such laws" [22, p. 124].

10.2. **Creation through a process of self-referral.** In Section 7, we saw that fractals demonstrate the process of creation through self-referral. This process is present everywhere in art—art is the product of the artist's inner life.

Art begins within the self-interacting reverberations of the artist's consciousness referring to itself alone. The interaction of these reverberations with the subjective impulses of the artist's feelings is brought to life on the surface of the canvas.

In this way, art is naturally and inevitably a self-referral process and depends on the self-interacting dynamics of the consciousness of the artist. As Maharishi (cited in [10, p. 332]) puts it:

> The artist comprehends the outlines of the figure—maybe a long face or a short nose—in his consciousness, and then he wants to depict it on marble, on paper, on clay, or on wood; he carves the wood, but he carves the wood to match the picture he contains in his awareness.

The self-portrait is a very concrete example of self-referral in art; the artist gives visual expression to feelings about his or her own self. The process of self-referral is also evident in an individual artist's development, how themes in earlier works of art are developed, refined, and matured in later works.

Besides fractals, there are many other examples of self-referral or self-interaction in mathematics. The symmetry transformations of a symmetric design or pattern give a self-interacting dynamics, showing how some parts of the design or pattern are the same as other parts. The theorems of Euclidean geometry are the result of the self-interacting dynamics of the axioms. Topology depends on the interaction of open sets.

10.3. **The full range of life from silence to dynamism.** The full range of life extends from dynamic activity to deep silence. Outer, relative life is active, ever-changing, and dynamic. Thoughts, feelings, and intuitions of the mind are less active. Underlying these levels is the field of pure consciousness, experienced during the Transcendental Meditation technique as non-active, unchanging, and silent. An artist who wants to capture the full value of life must capture this range into a work of art if the work is to be fulfilling, because, as Maharishi points out: "Extreme dynamic value, extreme silent value, both together make art—make the action waves of bliss, waves of bliss" [24, p. 322].

Artists fully recognize the importance of the presence of both silence and dynamism in art. Kasimir Malevich discusses harmonizing the opposite values of dynamism and silence in *The Non-Objective World*:

> Life wishes not to live but to rest—it strives not for activity but for passivity. For this reason, agreement among the dynamic or static values of the additional element affecting the system is taken for granted, and a "bringing-into-agreement" of the dynamic elements—systematizing them, that is—amounts to transforming them into static elements, for every system is static (even when it is in movement), whereas every construction is dynamic because it is "on the way" toward a system. [38, p. 14]

How skillfully an artist can integrate, harmonize, or "bring into agreement" these opposite values—living and resting, dynamic and static—into the wholeness of a work of art determines how great an impact the work will have on the viewer. Artists do this in different ways. L. Hilberseimer contrasts the work of Kasimir Malevich with that of Piet Mondrian in terms of how each deals with dynamism and silence [38, p. 8]: "Malevich's color concept was static but his concept of form, on the other hand, was dynamic. This stands in sharp contrast to the Neo-Plasticism of Piet Mondrian, in which the forms are static while the colors constitute the dynamic element."

Wassily Kandinsky starts his discussion of the elements of art [32] with recognizing the point as "the proto-element of painting." He views "the geometric point as the ultimate and singular **union of silence and speech**" [32, p. 25], and from that union, he sees the whole of art emerging.

The range of mathematics also is from dynamism to silence. Each area of mathematics has specific kinds of dynamism—transformations or functions—and specific kinds of silence or non-change—the invariants of the transformation. And, as in other areas of life, it is the invariants that give greater power and understanding.

10.4. **Life is structured in layers.** All of the physical world around us is structured in layers, from the galaxies, to the solar system, to our planet, to individual plants and animals, to organ systems, to molecules, atoms, subatomic particles, and finally to the unified field at the basis of the more expressed levels of life. The subjective world of the artist or mathematician is also structured in layers, from the senses, to the mind, to the emotions, to the intellect, to the ego, and finally to the field of pure consciousness or Being, experienced as the Self of an individual. Maharishi describes the qualities of the field of Being in this way:

> Underneath the subtlest layer of all that exists in the relative field is the abstract, absolute field of pure Being which is unmanifested and transcendental. It is neither matter nor energy. It is pure Being, the state of existence.
>
> This state of pure existence underlies all that exists. Everything is the expression of this pure existence or absolute Being which is the essential constituent of all relative life. [36, p. 5]

So, too, a work of art is structured in layers. An effective work of art can lead the viewer's awareness from the superficial, surface level of the work, to deeper levels, to the transcendental level, as Maharishi brings out:

> The nature of life, being expressive, being progressive, unfolds the inner values of life, and this is precisely what art is—the expression of fuller values of life. [24, p. 198]

Deeper levels of life are more powerful, as we can see by comparing power at the molecular level released by burning and power at the atomic level released in a nuclear reactor.

In mathematics, deeper levels are those that are more abstract, those that are more inclusive and more universal. Geometry uncovers patterns and relationships that depend on measurement of length, angle, and area. Topology is more abstract and, as we have seen, more powerful, locating patterns and relationships that are subtler than those of geometry, relationships that are unchanged when a shape is distorted by stretching or shrinking.

An artist must be able to encompass this full range of life subjectively in order to express it in the physical creation of art. When an artist can lead the viewer to this unbounded level of life, the purpose of art is achieved. We see an example of this in *Liberation* by M.C. Escher, Figure 26, which shows free-flying birds evolving from a pattern of triangles. Here, the symmetric tiling of patterns has the simplicity and abstraction of the unbounded field of consciousness, but gives rise to the physical diversity of a flock of birds.

A work of art makes an initial sensory impression on the viewer; this includes the shapes, colors, and content of the picture. If, as some report when viewing art, an experience of wholeness or transcendence occurs, the artist has been able to truly enliven the subtlest, deepest level of the viewer's consciousness.

11. Conclusion

Our exploration of geometry and art has uncovered many deep connections between them and with the science of consciousness. With an understanding of geometry, we can more thoroughly analyze the structure of a work of art and see how the artists' expressions convey what they see in their world. Making connections between the qualities of consciousness and geometry on the one hand to the structure of a work of art on the other hand leads the viewer to a greater appreciation of the wholeness of the work.

Teaching geometry in the context of connections to consciousness and its applications in art makes geometry relevant to students. *Geometry for the Artist* is a popular course at Maharishi University of Management, and students often use what they have learned in this course later in their artwork.

I hope that this paper has been able to demonstrate the effectiveness of the holistic interdisciplinary approach of Consciousness-Based education. I will leave the final words to the students themselves:[14]

- This course was extremely helpful in understanding how math reflects the structure of creation because I got a glimpse at how artists create worlds on a small canvas, and it began to unlock realization of how creation uses the same principles on a much smaller, yet massively larger scale. ... I think math in terms of art (creation) is how many of the great wonders,

[14] All comments from students who took the course in September, 2015, are given in Appendix A.

FIGURE 26. *Liberation* by M.C. Escher, 1955. Lithograph

inventions, and architecture of the Renaissance were created; the artists understood it on a much deeper level.

- *Geometry for the Artist* goes beyond the analysis of two disciplines interacting with each other. In this course we study the most infinite nature of art and of geometry. We use this exploration to find the geometrical nature of infinity in art. And, how that geometry can be seen as the basis for the expression of infinity that art can have.
- Art has always been a dominantly intuitive area for me. I see a work of art and get a sense of it. When I try to intellectualize it too much I find myself without the proper language to describe it. Using both geometry and Consciousness-Based education in this class helped me to integrate intellect and intuition and gave me a language to do so that seems more authentic than contrived.

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Appendix A: Student Comments

Given below are all student responses to the question "Please add any comments you might have about Consciousness-Based Education, particularly with reference to this course" given to the students of *Geometry for the Artist* in September 2015. They have been lightly edited for clarity.

Student A: Very well done in this course of showing the infinite value in shapes and how artists and mathematicians are very much the same in their expression. I like how we always related the lessons to Maharishi Vedic Science. I have a totally different view toward math thanks to your class.

Student B: Consciousness-Based education is the only way to go! It is the only way to feel fulfilled while within a classroom setting. Connecting math to art makes it seem a little more meaningful to those who could care less about math, as people wish to learn things in how it pertains to their own life.

Student C: With a course like this, learning how unbounded consciousness is the source of all art expression and geometry has opened my eyes to the boundless potential for my own self-expression.

Student D: This course was extremely helpful in understanding how math reflects the structure of creation because I got a glimpse at how artists create worlds on a small canvas, and it began to unlock realization of how creation uses the same principles on a much smaller, yet massively larger scale. Math has never seemed more practical than when taught in regards to artistry. Before, any math besides arithmetic and basic algebra seemed useless in my life, but now I see I am surrounded by it. I think math in terms of art (creation) is how many of the great wonders, inventions, and architecture of the Renaissance were created; artists understood it on a much deeper level.

Student E: *Geometry for the Artist* goes beyond the analysis of two disciplines interacting with each other. In this course we study the most infinite nature of art and of geometry. We use this exploration to find the geometrical nature of infinity in art and how geometry can be seen as the basis for the expression of infinity that art can have. In Consciousness-Based education we develop our own infinite nature through Transcendental Meditation. We also go beyond learning objective knowledge and use our subjective experience as an equal tool for gaining knowledge. When I can find myself in the objective knowledge I am gaining an infinite relationship with it.

Student F: Art has always been a dominantly intuitive area for me. I see a work of art and get a sense of it. When I try to intellectualize it too much I find myself without the proper language to describe it. Using both geometry and Consciousness-Based education in this class helped me to integrate intellect and intuition and gave me a language to do so that seems more authentic than contrived. I appreciated that we were encouraged to make these connections in the classroom context instead of making these connections outside of the classroom and not having space to express these things in the classroom. Consciousness-Based education offers a language to describe underlying aspects of consciousness intellectually, and also, through Transcendental Meditation, allows for me to become more sensitive and attuned to these patterns/flavors/aspects of consciousness and how consciousness moves in my own life. This was supportive to me finding authentic and meaningful connection to geometry and geometry as it related to art. In high school, I struggled the most with geometry of all math classes. I relied on rote memorization, and was rewarded for that with an "A" grade, but made no meaningful and lasting connection. This class and the context it was taught in actually facilitated a connection and relationship to the knowledge.

Student G: When education is based in consciousness, it means something. I have never had an easy time in math class before taking *Geometry for the Artist*. However, I have always viewed math as an entity separate from my own Being, and an evil entity at that. Dr. Gorini introduced math to me as an expression of the infinite. Math, like me, is part of the universe in ecstatic motion, so I should try to value it as such. This course has been a breeze and so much fun. Consciousness gives life to knowledge and a sense of purpose to daily activities, including school and homework.

Appendix B: List of Course Topics

Below is a list of the eighteen lessons of the course along with their science of consciousness themes.

Lesson 1: Geometry and Art: From Point to Infinity
Lesson 2: Classifying Symmetric Designs: Locating Nonchange within Change
Lesson 3: Classifying Band Ornaments: Locating Nonchange within Change
Lesson 4: Classifying Tilings: Locating Nonchange within Change
Lesson 5: Symmetry in the Work of Escher: Unbounded Creativity
Lesson 6: Linear Perspective: Connecting Knower and Known
Lesson 7: Checkerboards in Perspective: Pure Knowledge has Organizing Power
Lesson 8: Circles in Perspective: Harmony in Natural Law
Lesson 9: Two-Point and Three-Point Perspective: The Full Range of Creation
Lesson 10: Perspective in the Work of Escher: Expressing Inner Experience
Lesson 11: Similarity and Proportion: Unifying Differences
Lesson 12: Pictorial Composition: Knowledge has Organizing Power
Lesson 13: Fractals: The Part Contains the Whole
Lesson 14: Dynamical Systems and Chaos: Creation through Self-Referral
Lesson 15: The Mandelbrot Set: From Point to Infinity
Lesson 16: Lines, Curves, and Curvature: Creating Dynamism from Silence
Lesson 17: Non-Euclidean Geometries: Consciousness as a Field of All Possibilities
Lesson 18: Topology: Creating from the Home of All the Laws of Nature

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MATHEMATICS OF PURE CONSCIOUSNESS

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ABSTRACT. Adi Shankara, the foremost exponent of Advaita Vedanta, declared "Brahman alone is real, the world is mithya (not independently existent), and the individual self is nondifferent from Brahman." A fundamental question is, How does the diversity of existence appear when Brahman alone is? The *Yoga Vasistha* declares, "The world appearance arises only when the infinite consciousness sees itself as an object." Maharishi Mahesh Yogi has elaborated on this theme: Creation is nothing but the dynamics of pure consciousness, which are set in motion by the very fact that pure consciousness is conscious; being conscious, it assumes the role of knower, object of knowledge and process of knowing. To help clarify these issues, we offer a mathematical model of pure consciousness. We show that in a natural expansion of the universe of mathematics by ideal elements, there is a unique set Ω whose only element is itself, and which is equal to the set of all possible transformations from itself to itself. All "real" mathematical objects can be seen to arise from the internal dynamics of Ω. All differences among numbers, and among all mathematical objects, are seen to be ghostly mirages, hiding their true nature as permutations of one set, Ω.

1. INTRODUCTION

Adi Shankara, the foremost exponent of Advaita Vedanta, declared "Brahman alone is real, the world is mithya (not independently existent), and the individual self is nondifferent from Brahman."[1] How does the apparent diversity of existence arise when Brahman alone is? The *Yoga Vasistha* [16] declares,

> The world appearance arises only when the infinite consciousness sees itself as an object. (p. 357)

Maharishi Mahesh Yogi [8, 9, 11] has elaborated on this theme by first observing that pure consciousness, the singularity, by virtue of being consciousness, is in fact *conscious* and therefore conscious of itself. Therefore, because it is *consciousness*, pure consciousness assumes the role of knower and object of knowledge. Moreover, the process of observing, perceiving, and knowing is itself the activity of consciousness, the activity of pure consciousness knowing itself. It is by virtue of the self-interacting dynamics of pure consciousness knowing itself that there is a sequential unfoldment, an unmanifest dynamism, that appears on the surface to be our manifest universe.

In this paper, we attempt to investigate these unmanifest dynamics using the tools of modern set theory. Providing a mathematical model of pure consciousness

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[1] The transliterated Sanskrit is *Brahma satyam jagat mithya, jivo brahmaiva naparah.* This is a quotation from one of Shankara's famous works, *Vivekacudamani* or *Crest Jewel of Discrimination.* See [14, 67–68]. The translation given here comes from the Wikipedia article surveying the life and work of Shankara: `http://en.wikipedia.org/wiki/Adi_Shankara`.

and its dynamics makes it possible to bring clarity to the mystery of creation, which, in the Vedantic view, is only an appearance.

We will see that the mathematical universe is like the material universe in that it is composed of a vast array of distinct individuals, interacting according to laws of the universe. We will identify an "ideal element" Ω of the universe that uniquely exhibits characteristics and dynamics that parallel those of pure consciousness. We will then be in a position to see the sense in which all mathematical objects are in reality nothing but Ω, but, because of a "mistake of the intellect," all sets are seen to be distinct and unconnected to their source.

We develop our thesis by first reviewing key elements of Advaita Vedanta. We take as our source for this knowledge expressions from the *Yoga Vasistha* [16], one of the most important scriptures of Vedantic philosophy [15, p. 37ff.], and the elaboration on these and other parts of the Vedic literature provided by Maharishi Mahesh Yogi, who has made the lofty heights of philosophy and wisdom of Advaita accessible to the common man through simple procedures of meditation and a scientific approach to study of Veda [8, 9, 11].

We then review the structure of the mathematical universe as it is understood today in modern foundational studies. Resources for this treatment include [6] and [1]. In studying the mathematical universe, we will discover a partial analogue to the field of pure consciousness and its dynamics. By observing how this analogue fails to fully capture the dynamics of consciousness, we are then led to the possibility of expanding the universe to include an ideal element, which could more fully embody these dynamics. Having located such an element, Ω, we then show how it provides the key to recognizing the deeper truth about every standard mathematical object as being nothing other than Ω; at the same time, we will be able to give an account of the origin of the apparent separation of all mathematical objects from their source.

Most of the mathematical results mentioned in the paper are known. We have contributed a few new insights to the existing body of knowledge that support the coherence and cogency of our mathematical model. Seeing the realizations about the ultimate nature of reality modeled in this mathematical context will provide, we hope, a taste of this higher vision of life.

2. The Nature of the Singularity, Pure Consciousness

As we discussed in the Introduction, a key insight into the question, How does diversity arise from One? is that the "One," the singularity, is *pure consciousness*. By virtue of being consciousness, pure consciousness, the singularity, *automatically* assumes the roles of knower, known, and process of knowing. In Maharishi's treatment, the knower is referred to as Rishi, the known as Chhandas, and the process of knowing as Devata. Since the process of knowing has an impact on both the knower and the object of knowledge, this value of Devata is also to be appreciated as the principle of *transformation*. When these three are seen as one, they are referred to as Samhita (of Rishi, Devata, and Chhandas); Samhita in this context means *unity*.[2] The dynamics of pure consciousness can be seen here to be the dynamics

[2]Traditionally, each hymn in Rik Veda specifies the seer who saw (or heard) the hymn—Rishi; the meter of the hymn—Chhandas; and the Devata or impulse of intelligence that is being expressed in the hymn. (See [4].) The connection to Maharishi's use of these terms should be

by which pure consciousness knows itself and interacts with itself. This first step of diversification shows how unity can appear diversified without ever stepping out of itself, without ever really becoming anything other than One. In this section, we bring into focus aspects of these self-referral dynamics as they are expressed in the *Yoga Vasistha* and in Maharishi Vedic Science.

One theme in the dynamic unfoldment of pure consciousness within itself is the idea that, in knowing itself, in perceiving itself as an object, pure consciousness becomes as if focused on a point within itself, which the *Yoga Vasistha* [16] describes as a *seed of ideation*:

> My son, when, in the infinite consciousness, the consciousness becomes aware of itself as its own object, there is the seed of ideation. (p. 190)

In Maharishi's [11, pp. 171–174] treatment, the dynamics by which the infinitely expanded value of pure consciousness collapses to a point are displayed in the first syllable of Rik Veda: "AK." The letter "A" is uttered with an open voice, making an unrestricted sound, whereas the sound "K" represents a *stop* in the flow of sound; in this way the transformation of "A" to "K" expresses the collapse of the unbounded value of the singularity to a point. Maharishi [11, p. 171] explains that Rik Veda itself describes its own structuring dynamics; according to this description, the fundamental impulses and vibrational modes that arise in the process of pure consciousness knowing itself—the very structuring impulses of knowledge itself, of Veda itself—emerge in this collapse of "A":[3]

> *Richo akshare parame vyoman*
> The hymns of the Veda emerge in the collapse of "A", the "kshara" of "A".
>
> —Rik Veda 1.164.39

The dynamics indicated by the syllable "AK," representing the collapse of unboundedness to a point, are the dynamics inherent in pure consciousness, in *Atma*.[4] The unfoldment of the Veda and Vedic literature from the first syllable AK is likewise, therefore, an elaboration of dynamics hidden within *Atma* [8, pp. 500–503].

Maharishi [10, p. 4] also points out that, as any kind of knowledge has *organizing power*—power to yield material consequences and effects—so likewise must the most concentrated knowledge, pure knowledge, have maximum, *infinite*, organizing power. Therefore, he concludes, from the Veda and its infinite organizing power arises all of manifest existence [9, p. 409]. From these observations, he concludes that Veda and *Vishwa* are [9, p. 409] "the inner content of *Atma*."

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clear in the case of Rishi; for Chhandas, the meter has to do with the objective *structure* of the hymn, rendering fine impulses of intelligence as concrete form; and Devata is what links the Rishi to the hymn (Chhandas). Samhita is usually translated as "collection"; Maharishi translates Samhita without introducing the notion of division or separation: What must be true of Rishi, Devata, and Chhandas, from the viewpoint of Vedanta, is that, in being collected together (in the form of hymns), they are in reality *one*—just dynamics of pure consciousness. In Maharishi's treatment, therefore, in his translation of the word "Samhita," the unified aspect of "collection" is emphasized.

[3] Translation by Maharishi. See for example [8, p. 482].

[4] *Atma* is understood to be the unbounded, unlimited nature inherent in individual awareness. "*Jiva*, then, is individualized cosmic existence; it is the individual spirit within the body. With its limitations removed, *jiva* is *Atma*, transcendent Being" [13, p. 98].

The dynamics of unfoldment from *Atma* to Veda to *Vishwa* have another important characteristic: Each impulse that arises, each expression that emerges, remains connected to its source. Nothing that arises in this process of unfoldment is separate from pure consciousness. The *Yoga Vasistha* [16] explains it in this way:

> Thus the pure consciousness brings into being this diversity with all its names and forms, without ever abandoning its indivisibility, just as you create a world in your dream. (p. 638)

Indeed, each step of unfoldment is nothing other than transformations within pure consciousness itself; in the language of the *Yoga Vasistha* [16],

> The ignorant regard this samsara as real. In reality it does not exist at all. What does exist is in fact the truth. But it has no name! (p. 528)

We also read,

> It is only in the state of ignorance that one sees a snake in the rope, not in an enlightened state. Even so, to the enlightened vision, only the infinite consciousness exists, naught else. (p. 134)

This phenomenon—that we see undifferentiated pure consciousness as being a diversified manifest material universe—is referred to in Vedanta [14] as *vivarta*. Indeed, from this perspective, *vivarta* is responsible for each step of the apparent diversification of the singularity: from the analysis of one—*Atma*—into three (knower, known, process of knowing) and the appearance of the point value of pure consciousness within itself, to the emergence of impulses of self-knowing and the structuring of the Veda, to the appearance of the universe—each step in the process arises by virtue of this principle of *vivarta* [8]:

> Here, Unity (Samhita) appears to be diversity (Rishi, Devata, and Chhandas). This is the absolute eternal principle of *vivarta*, where something appears as something else. The very structure of knowledge (Samhita) has the principle of *vivarta* (Rishi, Devata, Chhandas) within it. (p. 589)

Also, we read,

> The principle of *vivarta* makes the unmanifest quality of self-referral consciousness appear as the Veda and Vedic Literature, and makes the Veda and Vedic Literature appear as Vishwa. (pp. 377, 589)

According to Maharishi, for the enlightened vision, for the knower of Brahman, the diversification that we see as the manifest universe is appreciated in terms of the one reality, wholeness, pure consciousness. Differences are seen but are as if transparent; what dominates is Unity. Summarizing such points made by Maharishi in conversation with Vernon Katz, Katz writes [7]:

> [In Unity Consciousness] the boundaries do not disappear ... only they cease to dominate. Where before they were opaque ... they are now fully transparent. (p. 47)

The principle of *vivarta* is also responsible for the apparent reality that the world is different from, separate from, pure consciousness; that things really are separate and not connected to each other or to a fundamental source. This perspective

Maharishi calls *pragya-aparadh*—mistake of the intellect. Quantum field theorist John Hagelin elaborates on this point in Maharishi Vedic Science [5]:

> Hence the notion of diversity disconnected from unity is a fundamental misconception. This misconception is known as pragya-aparadh or "mistake of the intellect." Pragya-aparadh results when, in the mechanics of creation from the field of consciousness, the intellect loses sight of the essential unity which is the true nature of the self ... The intellect gets caught up in its own creation, i.e., gets overshadowed by the perception of diversity to the exclusion of the unity which is the actual nature of the self being discriminated. According to Maharishi, this mistake of the intellect is so fundamental to the nature of human experience that it is responsible for all problems and suffering in life. (p. 284)

3. Locating the Singularity in the Mathematical Universe

Our goal in this section is to examine to what extent the vision of Advaita can be modeled within the standard foundation of mathematics, ZFC set theory. It is reasonable to attempt to find such a model for several reasons. First, the universe of mathematics resembles, in several important ways, the material universe, in that it contains "everything" and consists of apparently distinct individuals that interact in endless ways. Secondly, as we describe in more detail below, there is a natural analogue to the "singularity" within the standard foundation, namely, the *empty set*—the set having no elements. As we will see, every mathematical object is built up from the empty set, and it is possible to locate within every mathematical object its "origin" in the empty set.

We will show that, while this model, using the empty set as an analogue to pure consciousness, does capture some of the relationships that have been identified as principles and dynamics of pure consciousness, it falls short in a number of important ways. For example, we will not find that this singularity is fundamentally self-interacting or "three-in-one" by nature. And we will find that the differences among mathematical objects are rigid; the unity that we are able to locate, though significant, is sufficiently hidden to prevent this unity from being a dominant characteristic of mathematical objects. Having identified these shortcomings, we will be in a position to significantly improve our model in the next section.

We begin with a brief introduction to modern mathematical foundations. At the beginning of the twentieth century, modern mathematics became *one subject*; all the different fields of mathematics were at last seen to be limbs of a single tree of knowledge, the single field of *mathematics* [6]. This recognition can be described in three parts:

(1) *The recognition that every mathematical object can be represented as a set.* For instance, an ordered pair (a, b) can be represented as the set $\{\{a\}, \{a, b\}\}$. A function $f : A \to B$ can be represented as the set of ordered pairs $A_f = \{(x, y) \mid y = f(x)\}$. Whole numbers $0, 1, 2, \ldots$ are represented, respectively, by $\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}, \ldots$, where $\emptyset$ denotes the empty set, and each successive set y in the list is obtained from the previous x by the rule: $y = x \cup \{x\}$.

(2) *The introduction of a standard set of axioms.* These axioms are called the *Zermelo-Fraenkel axioms with the Axiom of Choice,* or ZFC. Every theorem in mathematics can be restated in terms of the language of sets and derived directly from the axioms of ZFC. Here are three examples of ZFC axioms:
Axiom of Empty Set. There is a set with no element.
Axiom of Pairing. For any sets X, Y there is a set whose only elements are X, Y (denoted $\{X, Y\}$).
Power Set Axiom. For any set X there is a set, denoted $\mathcal{P}(X)$, whose elements are precisely the subsets of X.

(3) *The universe V.* The universe V consists of all possible mathematical objects, represented as sets. The ZFC axioms "describe" how to build the universe V in stages $V_0, V_1, V_2, \ldots$. The zeroth stage V_0 is defined to be the empty set $\emptyset$. Each subsequent stage is obtained from the previous stage by an application of the *power set operator* $\mathcal{P}$. By definition, for any set A, $\mathcal{P}(A)$ is the set consisting precisely of the subsets of A. So, for example, $\mathcal{P}(\{a, b\}) = \{\emptyset, \{a\}, \{b\}, \{a, b\}\}$. Since the only subset of $\emptyset$ is $\emptyset$ itself,

$$V_1 = \mathcal{P}(V_0) = \mathcal{P}(\emptyset) = \{\emptyset\}.$$

Therefore, when we repeatedly apply the power set operator to the empty set, we obtain the stages of V as shown in Figure 1.

$$\begin{array}{lcl} V_0 & = & \emptyset \\ V_1 & = & \mathcal{P}(V_0) = \{\emptyset\} \\ V_2 & = & \mathcal{P}(V_1) = \{\emptyset, \{\emptyset\}\} \\ V_3 & = & \mathcal{P}(V_2) = \big\{\emptyset, \{\emptyset\}, \{\{\emptyset\}\}, \{\emptyset, \{\emptyset\}\}\big\} \\ \cdot & = & \cdot \\ \cdot & = & \cdot \\ \cdot & = & \cdot \end{array}$$

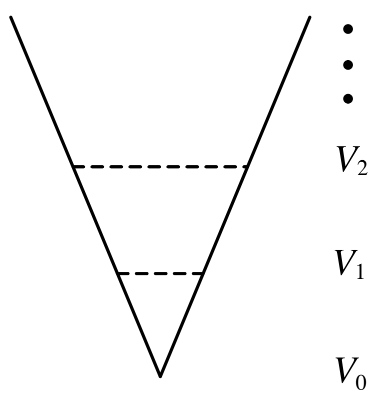

Figure 1. The Universe of Sets

If we display the elements of V in its first few stages[5] (Figure 2), we observe an interesting pattern: Every set in the universe is, at its "core," just the empty set; every set is represented syntactically by a sequence of curly braces and empty set symbols. One could say that every set is just another way of looking at the empty set.

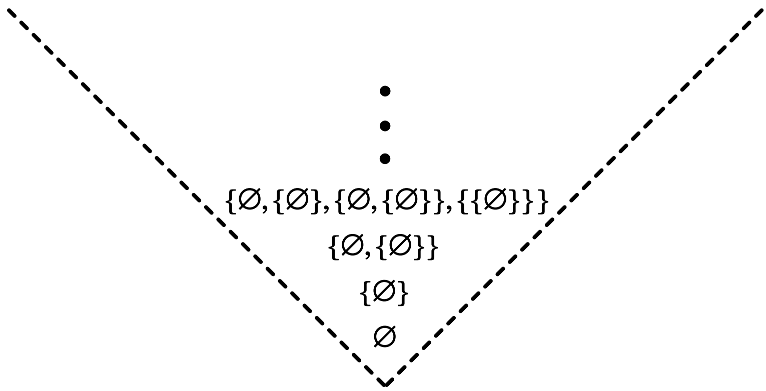

FIGURE 2. V As Permutations of the Empty Set

If we consider the empty set as a representation of the singularity, pure consciousness, then Figure 2 provides, to some extent at least, a model of the principle that pure consciousness "pervades everything inside and out" [16, p. 513]. Moreover, it is always possible to locate this "transcendental value" of any set in a finite sequence of steps, as we now describe. We first make several observations about sets in the universe [6]:

(1) By virtue of the construction of V, every nonempty set belonging to V is composed of elements that are themselves sets.

[5]We have not given the full definition here for the sake of simplicity. A precise formulation requires the use of *infinite ordinal numbers*. Roughly speaking, the infinite ordinals extend the whole numbers, permitting enumerations of infinite sets of different sizes. Infinite ordinals are like whole numbers except that some of them do not have immediate predecessors. For example, if we let ω be the first infinite ordinal, the "number" that comes immediately after all the whole numbers, then ω has no immediate predecessor, whereas the number 5, for example, does have an immediate predecessor, namely 4. Ordinals with no immediate predecessor are called *limit ordinals*. The collection of all ordinals, including the usual whole numbers, is denoted ON. The formal definition of the stages of V is given by the following clauses:

$$\begin{aligned} V_0 &= \emptyset \\ V_{\alpha+1} &= \mathcal{P}(V_\alpha) \\ V_\lambda &= \bigcup_{\alpha<\lambda} V_\alpha \qquad (\lambda \text{ a limit ordinal}) \\ V &= \bigcup_{\alpha\in \mathrm{ON}} V_\alpha . \end{aligned}$$

(2) Every set X has a *rank*, which signifies the least stage in the construction in which X occurs as a subset. For instance, $\{\emptyset\}$ is a subset of V_1 but not of V_0, so $\text{rank}(\{\emptyset\}) = 1$. Likewise, $\{2\}$ can be shown to be a subset of V_3 but not of V_2, and so $\text{rank}(\{2\}) = 3$.

(3) For every X in the universe and for every $x \in X$, $\text{rank}(x) < \text{rank}(X)$.

(4) *Infinite* $\in$*-chains* do not exist in V; that is, there do not exist sets x_0, x_1, x_2, ... in the universe for which the following holds:

$$\cdots \in x_2 \in x_1 \in x_0.$$

To see this, suppose such an infinite $\in$-chain $\cdots \in x_2 \in x_1 \in x_0$ does exist; we call x_0 the *starting point* of the chain. Now pick such a chain whose starting point has the least possible rank. We denote this chain $\cdots \in y_2 \in y_1 \in y_0$. But now $\cdots \in y_3 \in y_2 \in y_1$ (removing y_0 from the list) is also an infinite $\in$-chain whose starting point y_1 has rank less than $\text{rank}(y_0)$ (since $y_1 \in y_0$), and this contradicts the leastness of y_0.

The property (4) is expressed by saying that every set in V is *well-founded*: No set can be the starting point of an infinite $\in$-chain. It is equivalent to one of the axioms of ZFC, the Axiom of Foundation.

We observe next that for any nonempty set X in the universe, at least one element x of X is an $\in$*-minimal element*; this means that, for any $y \in x$, no element of y belongs to X. So, for example (recalling that whole numbers $0, 1, 2, \ldots$ are represented as the sets $\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}, \ldots$), the set $X = \{1, \{2\}, 3\}$ has two $\in$-minimal elements, 1 and $\{2\}$: The only element of 1 is $\emptyset$, and it does not belong to X; and the only element of $\{2\}$, namely, 2, also does not belong to X. On the other hand, one of the elements of 3, namely, 1, *does* belong to X, and so 3 is *not* $\in$-minimal in X. In general, one may find an $\in$-minimal element of a given nonempty set X by noticing that any element x of X having least rank among all elements of X—that is, $x \in \{y \in X \mid \text{for all } z \in X, \text{rank}(y) \leq \text{rank}(z)\}$—must be $\in$-minimal in X.

Finally, given any nonempty set X in the universe, we show how to arrive at its "core" in finitely many steps: Let $x_0 = X$ and let $x_1 \in x_0$ be an $\in$-minimal element of x_0. If x_1 is empty, we stop the construction; otherwise, obtain an $\in$-minimal element x_2 of x_1. Again, if x_2 is empty, stop; otherwise obtain an $\in$-minimal element x_3 of x_2. This process leads to a possibly finite sequence of sets $\cdots \in x_2 \in x_1 \in x_0$. If the sequence is finite, by construction, the leftmost set must be the empty set, since the construction will terminate only if the next $\in$-minimal element selected is nonempty. But now we observe that the sequence *must* be finite in every case because, by (4), there are no infinite $\in$-chains in the universe. Therefore, we have extracted from X, in finitely many steps, the "core" of X, namely, $\emptyset$.

We have seen that the universe V, being the all-inclusive container of all diversity in mathematics, is a mathematical parallel for the manifest universe. The fact that all sets are built from a singularity—a set devoid of all content; the fact that all sets can in fact be seen as different ways of viewing this singularity $\emptyset$; and the fact that this source of all sets can be located deep within any set in finitely many steps—these facts show that, in certain respects, V, together with the empty set, model the Vedantic view that the manifest universe is permeated by pure consciousness.

However, there are aspects of the Vedantic view that have no counterpart in our mathematical model.

One aspect of the Vedantic view of pure consciousness that is missing from our model is the *dynamics* of consciousness itself. For one thing, our mathematical representative of the singularity has no internal characteristics that would lead to the emergence of analogues to "knower," "known," and "process of knowing."

On the other hand, we do find the principle "unboundedness collapses to a point" modeled partially: The dynamics by which $\emptyset$ is transformed into $\{\emptyset\}$ is a transformation from abstract emptiness to the objectification of that emptiness as the set $\{\emptyset\}$ which contains it; in a sense, then, the boundary-less value of emptiness, embodied in $\emptyset$, collapses to emptiness-with-boundary, embodied in $\{\emptyset\}$, as the universe of sets emerges in its stage-by-stage unfoldment. What is missing here is the fact that the point that appears within pure consciousness is in reality no different from pure consciousness itself. Recall from the *Yoga Vasistha* [16],

> When this understanding arises in one, though there is self-awareness, even that ceases for there is no division between the observer and the observed. (p. 513)

This reality is obscured in the first transition step from V_0 to V_1 in the construction of V. It is not the case that $\emptyset = \{\emptyset\}$. What we do see modeled here is the emergence of what Maharishi has called *pragya-aparadh*, mistake of the intellect, the notion of diversity disconnected from unity (see Section 2).

This separateness of parts and disconnection of parts from their source then is propagated as stages of V continue to emerge. The fact that differences dominate in the construction of V is obvious when one compares any distinct sets in V. For instance, consider $\{1, 3\}$ and $\{\{4\}\}$. There is no mathematical sense in which these sets can be seen as "fundamentally the same"; and the fact that they have a common source in $\emptyset$, while true, is not obvious, but is rather hidden from view. The result is that the construction of the universe departs from the self-referral unfoldment that occurs within pure consciousness by which everything emerges; what is created instead is a world of *concepts* rather than a progressive unfoldment of *reality*. Again, from the *Yoga Vasistha* [16], we read,

> Hence, O Rama, abandon all forms of division—division in terms of time or of parts of substance—and rest in pure existence. These divisions are conducive to the arising of concepts. (p. 319)

Maharishi makes the same point explicitly with regard to the way in which sets and mathematical structures unfold in the mathematical universe [8]:

> The reality of Samhita, which is not overshadowed by Rishi, Devata, Chhandas, is the marvel of Vedic Mathematics, unknown to the mathematics of the conceptual world, unknown to the mathematics of the world of diversity, which has its basis in the notion of reality—not the reality but the notion about it, the concept of it. (pp. 557–558)

To improve our model, one could try to replace the empty set, as a representative of pure consciousness, with a set x having the property that $x = \{x\}$.[6] However, it

[6]A similar point is made in [8, p. 628].

is easy to see that, because every set in V is well-founded, V contains no such set x: If it did, it would give rise to the following infinite $\in$-chain:

$$\cdots \in x \in x \in x.$$

Despite this theoretical restriction, it is possible to create an expansion of the standard universe of sets in which an element of this kind does indeed exist. Such a universe can be created in a way that is analogous to the way in which the number $i = \sqrt{-1}$ can be added to the usual set of real numbers to produce an expanded field, the field of complex numbers. The number i arises as a solution to a certain equation over the set $\mathbb{R}$ of reals, namely $x^2 + 1 = 0$. In like manner, an expansion of V can be created by introducing a solution to the equation $x = \{x\}$, together with a large number of other such equations that have this sort of self-referential flavor. In this expansion, there is a unique set, denoted Ω, that satisfies this equation: $\Omega = \{\Omega\}$. We will see in the next section that Ω provides a more suitable representative of the singularity in the realm of sets.

Our study of Ω in the next section will begin by introducing a new way of representing sets: via *directed graphs*. To give a flavor of what is to come, consider the set $X = \{0, 1\}$. We can consider the elements of X to be vertices of a directed graph in which, for any vertices x, y, we have an edge $x \to y$ if and only if $y \in x$. With this scheme, one way to depict X as a graph is as follows:

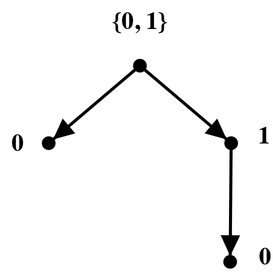

FIGURE 3. The Set $\{0, 1\}$ As a Directed Graph

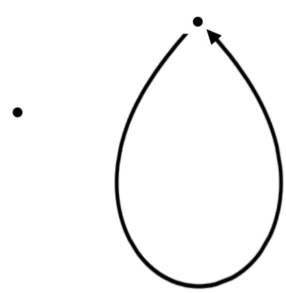

FIGURE 4. Representing $\emptyset$ and Ω As Directed Graphs

With graphs, we have a visual way to examine the structure of sets. This way of viewing sets leads to an illuminating example (Figure 4). The first graph represents the empty set; the second graph represents Ω. What is missing in the first graph, but present in the second, is a *self-referral relationship* of the set with itself. Whatever self-referral dynamics may be present *within* the empty set, speaking metaphysically, these dynamics are not accessible *mathematically*; we do not find self-referral dynamics within $\emptyset$. However, we do find such dynamics explicitly represented in the graph for Ω: Ω is related to itself—it itself is its only element. As we will show, structuring the universe on the basis of Ω rather than $\emptyset$ leads to a vision of sets that reveals the fundamental unity of all diversity in a very explicit way.

The fact that a self-referral loop is at the basis of this new model accords well with an insight expressed by Maharishi [11] regarding the structuring dynamics of pure consciousness:

> The evolution of consciousness into its object-referral expressions, ever maintaining the memory of its self-referral source—ever-evolving structure of consciousness maintaining the memory of its source—progresses in self-referral loops—every step of progress is in terms of a self-referral loop. (p. 64)

4. Unfoldment of the Universe Within the Unmanifest

In the previous section, we argued that the universe of sets, together with the empty set, models certain aspects of the unfoldment of pure consciousness, but fails to capture other aspects. One lack in this respect is that, although, in the construction of V, we find a "collapse" from the boundary-less and content-free set $\emptyset$ to a concrete single-element set $\{\emptyset\}$, what is lost is the connection of this expressed value to its source in $\emptyset$; we argued that this transition from $\emptyset$ to $\{\emptyset\}$ is the sprouting of *pragya-aparadh*, the beginning of separation between individuals and their source. We also mentioned that if, in place of $\emptyset$, we could obtain a set x that satisfies the equation $x = \{x\}$, such a set would give expression both to the dynamics of "collapse from unboundedness to a point" and also to the theme of *self-referral* dynamics, by which expressions remain connected to their source in their unfoldment.

We also observed that within pure consciousness emerge three values: the knower (Rishi), the known (Chhandas), and the process of knowing and transformation (Devata). This emergence of three from unity has no parallel in our analogy in the realm of sets, where sets emerge from the empty set. On the other hand, if there could exist a set x that not only satisfies the equation $x = \{x\}$, but also has the property that x is a transformation from itself to itself—that is, $x : x \to x$—then we would have given mathematical expression to two of these three: To the idea that x is an *object of knowledge* of itself, assuming the role of Chhandas, because of the relation $x = \{x\}$, and also to the notion that x is a *transformation* within itself, assuming the role of Devata, because of the fact that x is the map $x : x \to x$.

The dynamics of pure consciousness could be modeled even more profoundly if, in addition, x satisfies $x = x^x$, where x^x signifies the set of *all possible* transformations from x to itself. Notice that in that case, since $x \in x = x^x$, it follows that

$x \in x^x$ and so x is a transformation $x : x \to x$. Therefore, from these two equations, $x = \{x\}$ and $x = x^x$, we also give expression to the Chhandas and Devata values. But, because $x = x^x$, we can say more: Now, x includes within itself all possible transformations of itself; in this sense, it is the *witness* of all its internal transformations, and in that role, it is the knower, the Rishi. The connection between "witness" and "Rishi" is discussed in [5] in which Maharishi's approach to this topic is elaborated:

> In the structure of knowledge, Rishi is the knower—the lively, discriminative but unmanifest basis of knowledge, which stands as a witness to the known and process of knowing. (p. 255)

The discussion above suggests that, in order to capture more of the dynamics of pure consciousness than we have been able to achieve using the empty set as a model, it would be desirable to find a solution to the following equations:

$$x^x = x = \{x\}. \qquad (+)$$

A solution Ω to (+) would have the following characteristics:

(1) Ω consists precisely of all possible transformations of itself to itself.
(2) One of the transformations of Ω to itself is Ω itself: $\Omega : \Omega \to \Omega$ (since $\Omega \in \Omega^\Omega$).
(3) Ω and its collapsed value $\{\Omega\}$ are one and the same.

In this section we show there is a natural (and unique) mathematical solution Ω to the equations (+), using a slight expansion of the usual ZFC universe. We show also how it is possible to derive all sets in the universe from this one point Ω, and at the same time, how all sets naturally collapse back to Ω. Moreover, the entire mathematical landscape will be seen, in this view, to be nothing but patterns and permutations of this one "reality" Ω; differences between individuals will no longer dominate.

One other consequence of (+) that we mention here is that Ω embodies, in an abstract sense, the very dynamics of nature's functioning, of the functioning of the laws of nature. This can be seen by the fact that Ω is itself equal to the evaluation map $eval : \Omega^\Omega \times \Omega \to \Omega$ defined by $eval(f, p) = f(p)$. The evaluation map expresses the way in which natural law is applied to each point in existence to carry it forward to the next stage of its unfoldment. Each impulse of evolution can be represented by a function, a kind of transformation, and "points in the universe" correspond to sets. The evaluation map is, in this sense, the master plan for all evolutionary dynamics, governing the application of each evolutionary impulse f to each point p of the manifest universe, producing a new, "more evolved" value, $f(p)$.

We summarize the parallels we have identified so far between dynamics of consciousness and dynamics of Ω:

The Set Ω As Samhita of Rishi, Devata, and Chhandas, and Administrator of the Cosmos

(1) *Rishi:* $\Omega = \Omega^\Omega$
(2) *Devata:* $\Omega : \Omega \to \Omega$

(3) *Chhandas:* $\Omega = \{\Omega\}$
(4) *Structuring Dynamics of the Universe*:

$$\Omega = eval : \Omega^\Omega \times \Omega \to \Omega : (\Omega, \Omega) \mapsto \Omega.$$

We demonstrate the fact that

$$\Omega = eval$$

in the following Proposition. We take as our background theory the ZFC axioms without the Axiom of Foundation (denoted ZFC^-), together with the assumption that Ω is indeed a solution to the equation $x = \{x\}$. The proof will show that, once we know Ω is a solution to $x = \{x\}$, we can conclude without additional assumptions that it is a solution to $x = x^x$ as well. To draw further conclusions, we will assume somewhat more later on.

Proposition 1. (ZFC^-) *Suppose Ω is a solution to the equation $x = \{x\}$. Then the following statements hold true:*

(A) $\Omega = (\Omega, \Omega)$.
(B) $\Omega = \Omega \times \Omega = \Omega^\Omega = \Omega^\Omega \times \Omega$.
(C) $\Omega \in \Omega^\Omega$*, so we may write* $\Omega : \Omega \to \Omega$.
(D) *For all* $x \in \Omega$, $\Omega(x) = \Omega$ *(using the representation of* Ω *in* (C)).
(E) $\Omega = eval$.

Proof of (A). We have the following derivation (using the fact that, by definition, for any sets x, y, $(x, y) = \{\{x\}, \{x, y\}\}$).

$$\begin{aligned}
(\Omega, \Omega) &= \{\{\Omega\}, \{\Omega, \Omega\}\} \\
&= \{\{\Omega\}, \{\Omega\}\} \\
&= \{\{\Omega\}\} \\
&= \{\Omega\} \\
&= \Omega.
\end{aligned}$$

Proof of (B). For the first equality, we have:

$$\begin{aligned}
\Omega \times \Omega &= \{(x, y) \mid x \in \Omega \text{ and } y \in \Omega\} \\
&= \{(\Omega, \Omega)\} \\
&= \{\Omega\} \quad \text{by (A)} \\
&= \Omega.
\end{aligned}$$

To show $\Omega = \Omega^\Omega$, we compute as follows: Notice first that there is only one function $f : \{\Omega\} \to \{\Omega\}$, namely, the function f defined by $f(\Omega) = \Omega$. We denote this function f_Ω. Note that as a set of ordered pairs, $f_\Omega = \{(\Omega, \Omega)\}$. Therefore:

$$\begin{aligned}
\Omega^\Omega &= \{f \mid f : \Omega \to \Omega\} \\
&= \{f \mid f : \{\Omega\} \to \{\Omega\}\} \\
&= \{f_\Omega\} \\
&= \{\{(\Omega, \Omega)\}\} \\
&= \{\{\Omega\}\} \quad \text{by (A)} \\
&= \{\Omega\} \\
&= \Omega.
\end{aligned}$$

Finally, the fact that $\Omega \times \Omega = \Omega^\Omega \times \Omega$ follows from $\Omega = \Omega^\Omega$.

Proof of (C). This follows from the fact that $\Omega = \Omega^\Omega$, shown in (B), and the fact that $\Omega \in \Omega$, which follows from the fact that $\Omega = \{\Omega\}$.

Proof of (D). Since $\Omega = \{\Omega\}$, the only element of Ω is Ω. Therefore, it suffices to show that $\Omega(\Omega) = \Omega$. But this is equivalent to the assertion that $(\Omega, \Omega) \in \Omega$, and this follows from (A) and the fact that $\Omega \in \Omega$.

Proof of (E). The set-theoretic definition of *eval* is:

$$eval = \{(x, y, z) \in \Omega^\Omega \times \Omega \times \Omega \mid x(y) = z\}.$$

For any $(x, y, z) \in eval$, the fact that $(x, y) \in \Omega^\Omega \times \Omega$ implies that $(x, y) = (\Omega, \Omega)$; and the fact that $z \in \Omega$ implies $z = \Omega$. Therefore, the only element of $\Omega^\Omega \times \Omega \times \Omega$ is (Ω, Ω, Ω), and this element (x, y, z) does satisfy $x(y) = z$ (as shown in (D)). Therefore,

$$eval = \{(\Omega, \Omega, \Omega)\} = \{((\Omega, \Omega), \Omega)\} = \{(\Omega, \Omega)\} = \{\Omega\} = \Omega,$$

as required. □

Proposition 1 has been established in the theory ZFC$^-$ together with the assumption that there is a solution to the equation $x = \{x\}$—more formally, from the theory ZFC$^- + \exists x\, x = \{x\}$—but we have not yet demonstrated that this theory is consistent. We handle this issue by showing that existence of a solution to $x = \{x\}$ is provable from the theory ZFC$^-$ + AFA, where AFA stands for the *Anti-Foundation Axiom*, a well-known alternative to the Axiom of Foundation, due to Forti and Honsell [3] and popularized by P. Aczel [1]. It is known that if ZFC is consistent, so is the theory ZFC$^-$ + AFA. Later in this article, we will discuss some important points about the proof of this fact.

We give a quick introduction to this axiom AFA so that we can use it to gain additional insights into the model of pure consciousness that we have proposed. We begin with several definitions. A (directed) graph G is a pair (M, E) consisting of a set M of vertices and a set E of edges. Edges are represented by pairs of vertices. So, if u, v are vertices in a graph G and G has an edge from u to v, this edge is denoted (u, v). We also write $u \to v$ to indicate that $(u, v) \in E$.

A *pointed* graph is a graph with a designated vertex, called its *point*. When p is such a designated vertex for a graph $G = (M, E)$, we denote the pointed graph

(G, p) or (M, E, p). When we draw pointed graphs, the point of the graph is the top vertex (whenever that makes sense) and descendants of the point evolve downward. (See examples below.)

A pointed graph $(G, p) = (M, E, p)$ is *accessible* if for all $v \in M$ there is a path from p to v in G. Accessible pointed graphs are referred to by the acronym *apg.* If, for all v, there is exactly one path from p to v, then G is called a *tree.* A graph is *well-founded* if it has no infinite path.

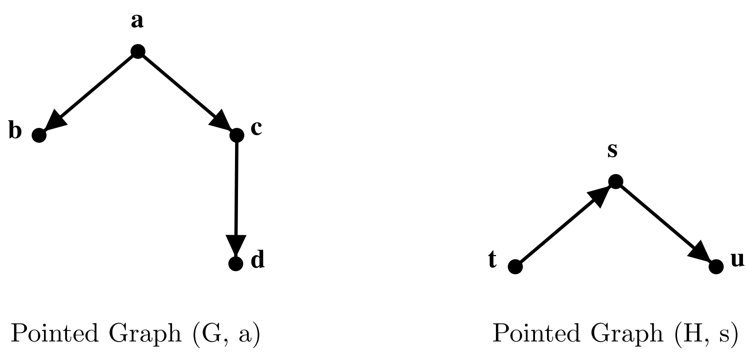

FIGURE 5. Examples of Pointed Graphs

In Figure 5, the left graph (G, a) has point a; it is well-founded and accessible. The right graph (H, s) has point s, but since there is no path from the point s to the vertex t, (H, s) is not accessible. Notice that if we change the point of H to be t, (H, t) is now an accessible pointed graph.

A *decoration* of a graph is an assignment of a set to each vertex of the graph in such a way that the elements of a set assigned to a vertex are always assigned to the children of that vertex. In symbols, a decoration of G is a map $d : G \to V$ such that, for all vertices v, w of G,

$$v \to w \text{ if and only if } d(w) \in d(v).$$

A *picture* of a set X is an apg that has a decoration in which X is assigned to the point.

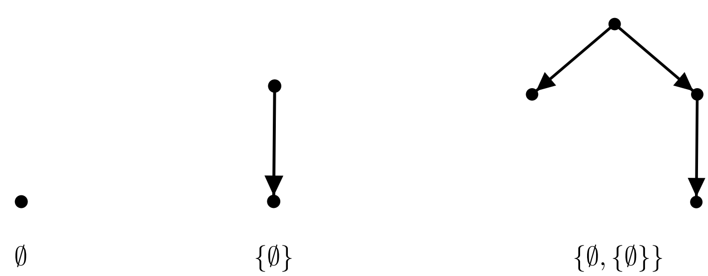

FIGURE 6. Pictures of Well-founded Sets

Figure 6 exhibits examples of well-founded apgs; each apg shown is a picture of the set that labels it. The first apg was introduced at the end of the last section. This graph has just one vertex and no edges; this means that the set it represents can have no elements. Accordingly, the unique set that is represented by this apg is the empty set $\emptyset$. The designated point for the second apg has just one child, which, in turn, has no children; accordingly, the set it represents is $\{\emptyset\}$, the set whose only element is $\emptyset$. Similarly, the third apg shown represents the set having as children the empty set and the set whose only element is the empty set, namely, $\{\emptyset, \{\emptyset\}\}$.

We state some important facts about representing the sets in a ZFC universe with graphs. The following theorem does not require AFA; it follows from ZFC:

Theorem 2.
(A) *Every well-founded graph has a unique decoration.*
(B) *Every well-founded apg is a picture of a unique set.*
(C) *Every well-founded set has a picture.*

The examples of Figure 6 illustrate Theorem 2(A); in these simple cases, it is easy to see that there is only one way to decorate the vertices of the given apgs with sets. A reasonable generalization of Theorem 2(A) to all possible apgs is the Anti-Foundation Axiom:

The Anti-Foundation Axiom (AFA). *Every graph has a unique decoration.*

An immediate consequence is the following:

Proposition 3. (Uniqueness Theorem) *Every apg is a picture of a unique set.*

A consequence of the Uniqueness Theorem is that the following apg uniquely determines a set:

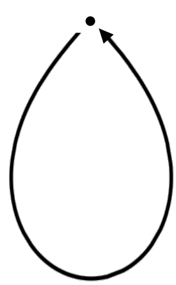

Figure 7. A Single-Loop Graph

The unique way to decorate this graph is with a set whose only element is itself; as we proved earlier, such sets cannot exist in any universe built from the standard ZFC axioms because existence of such sets contradicts the Axiom of Foundation. Since AFA contradicts the Axiom of Foundation, in order to work with AFA in a consistent way, we must remove the Axiom of Foundation from our basic set of axioms. In the theory ZFC^{-} + AFA, the Uniqueness Theorem does indeed hold true.

The unique set pictured by the single-loop graph of Figure 7 is usually denoted Ω. See Figure 8. We observe that, with regard to Figure 7, AFA tells us two things:

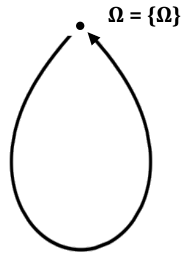

FIGURE 8. Single-Loop Graph with Unique Decoration Ω

First, that the single-loop graph is a picture for *some* set; and second, that there is *only one* set for which this graph is a picture. This latter point is as important as the first. Without AFA, even if we assume that the single-loop graph of Figure 7 can be decorated with a set $X = \{X\}$, there is no guarantee that it is unique (there could be X and Y such that $X = \{X\}$ and $Y = \{Y\}$ and $X \neq Y$).

In our work in this section so far, we have used the symbol Ω in two different ways: as a solution to $x = \{x\}$ and also as the unique decoration for the single-loop graph. We show now that this ambiguous usage is justified.

Theorem 4. (ZFC^- + AFA). *Let Ω be the unique decoration of the loop graph shown in Figure 7. Then Ω is the unique solution to the equations* (+)*; that is, Ω is the unique set for which $\Omega = \{\Omega\}$ and $\Omega = \Omega^\Omega$.*

Proof. The fact that $\Omega = \{\Omega\}$ follows from the structure of the apg for Ω: Certainly the designated point of the single-loop graph is a child of itself, so it follows $\Omega \in \Omega$. But it is also clear that the designated point is its *only* child. Therefore, Ω is the *only* element of Ω, and so $\Omega = \{\Omega\}$. By Proposition 1, $\Omega = \Omega^\Omega$ as well.

To see that Ω is the *unique* solution to the equations (+), note that any solution to these equations—even to the single equation $x = \{x\}$—is a decoration of the single-loop apg pictured above. By AFA, there is only one such decoration. Therefore, there is only one solution to (+). □

Theorem 4, together with the fact that $\mathrm{ZFC}^- + \mathrm{AFA}$ is consistent whenever ZFC is consistent, shows that ZFC^- is consistent with the statement $\exists x\, x = \{x\}$, and so our earlier work in this section is fully legitimized.

Before embarking on a discussion about how all real sets can be seen to arise from, and return to, the single non-well-founded set Ω, we spend some time exploring how a ZFC^- + AFA universe is built.

Building a $\mathrm{ZFC}^- + \mathrm{AFA}$ Universe

Let V denote the usual universe of sets—a model of ZFC—as discussed earlier. One can build, within V, a model $\hat{V}$ of ZFC^- + AFA. Moreover, any such model will have a well-founded part $\mathrm{WF} = \mathrm{WF}_{\hat{V}}$ (consisting of all the well-founded sets in $\hat{V}$) that is isomorphic to the original ZFC model V: $\mathrm{WF} \cong V$.

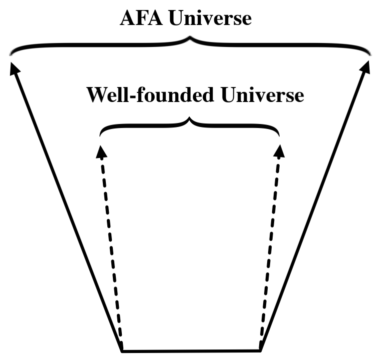

Figure 9. An AFA Universe Is an Expansion of the Well-founded Universe

In this way, we obtain the intuitive picture that a $\mathrm{ZFC}^- + \mathrm{AFA}$ universe is an expansion of the standard cumulative hierarchy of well-founded sets (see Figure 9)—an expansion consisting of well-founded sets together with *ideal elements*, which in the present context are the non-well-founded sets, like Ω.

We take a moment to describe, at a high level, how a universe for $\mathrm{ZFC}^-+\mathrm{AFA}$ can be constructed. We begin with the usual well-founded universe V of ZFC. Roughly speaking, we wish to think of the sets of our new universe as being precisely the apgs that live in V. This isn't quite right though because different (nonisomorphic) apgs can picture the same set, even in the well-founded case. For example, each of the apgs in Figure 10 (below) is a picture of the (well-founded) set $\{0, 1\}$, but the underlying graphs are nonisomorphic (since, for example, their vertex sets have different sizes).

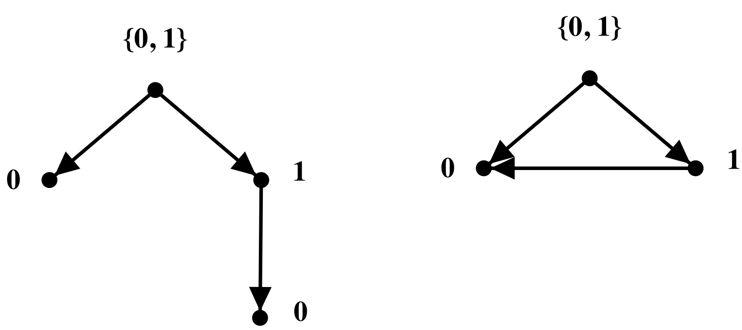

Figure 10. Nonisomorphic Pictures of the Same Set

This situation exactly parallels the situation one faces in attempting a construction of the reals from the rationals—a first try is to declare that a real is a Cauchy sequence of rationals. Just as many apgs may represent the same set, so likewise in the context of constructing the real line, we face the fact that many Cauchy sequences converge to the same real. The solution in the latter case is to form a quotient by an appropriate equivalence relation. In constructing the real line, one would like to declare two Cauchy sequences to be equivalent if they converge to the same real, but, since the reals have not yet been constructed, this approach cannot be used (though it serves to guide the intuition about it). Likewise, we would like to declare that two apgs are equivalent if they picture the same set, but since we have not yet constructed all the sets of our new universe, this statement of the equivalence relation is not formally correct. To capture this idea without assuming existence of the non-well-founded sets we are trying to build, the necessary equivalence relation, called *bisimilarity*, is formulated in another way. Ultimately, the universe that we build will consist of all the equivalence classes of apgs under the bisimilarity relation.

Since the definition of the bisimilarity equivalence relation is somewhat technical, we save a discussion of those details for the Appendix. For our discussion here, it will be enough to rely on the guiding intuition that two apgs are equivalent if they picture the same set, and we consider a couple of examples.

When apgs happen to be well-founded, we already know which sets they picture, because we are starting from the universe V of well-founded sets. As we observed before, the two apgs shown in Figure 10 picture the same set, namely, $\{0, 1\}$, and so they are bisimilar.

Figure 11 illustrates a second example. Here, each of the displayed non-well-founded apgs pictures the same non-well-founded set, Ω. Notice that each of the apgs shown *can be* decorated with Ω. But then by the uniqueness part of AFA, Ω is the *only* set that can decorate these apgs. Here again these apgs are bisimilar.

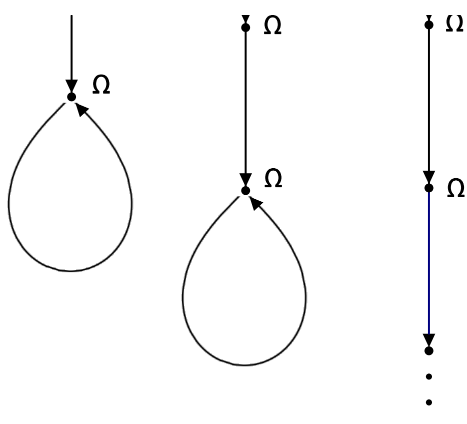

FIGURE 11. Pictures of Ω

Starting from the standard ZFC universe V, then, we build a subclass $\hat{V}$ consisting of these equivalence classes of apgs. For $\hat{V}$ to be a valid "universe of sets," it needs to have its own version of the membership relation. We describe this in an intuitive way here and give more details in the Appendix. The following example will illustrate the idea:

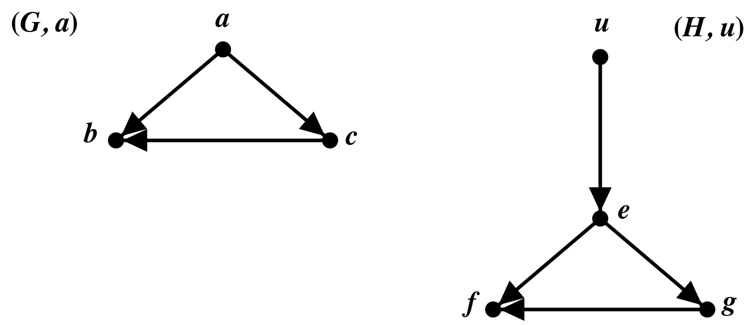

FIGURE 12. Membership in $\hat{V}$

In Figure 12, the (equivalence class of the) apg (G, a) is to be thought of as a "member" of the (equivalence class of the) apg (H, u) because, if we look at the sub-apg of H having point e—and we denote this subgraph He—then the apgs (He, e) and (G, a) are essentially the same (in this case, they are actually isomorphic); in addition, e itself is a child of the point u of H.

This example is typical: In general, (the equivalence class of) an apg (G, p) is a "member" of the (equivalence class of the) graph (H, q) if there is a child r of q—that is, we have $q \to r$—such that the sub-apg (Hr, r) is essentially the same as the apg (G, p) (here, "essentially the same as" means "bisimilar to").

The Ideal Elements of $\hat{V}$ As Solutions to Equations

Earlier in this article, we mentioned that a ZFC$^-$ + AFA universe could be viewed as an expansion of the usual class V of well-founded sets by adding "ideal" elements, like Ω, and that the procedure for forming such an expansion is analogous to adjoining the ideal, pure imaginary element i to the field $\mathbb{R}$ to obtain the complex field $\mathbb{C} = \mathbb{R}(i)$. In this subsection, we give an overview of how this can be done.

We recall that the pure imaginary number i arises as a solution to the equation $x^2 + 1 = 0$ over $\mathbb{R}$. We show that one may also view the expansion from V to $\hat{V}$ as arising from the introduction of "ideal" solutions to—in this case—*classes* of equations. One such equation, as we have seen, is $x = \{x\}$. Another is $x = (0, x)$. An example of a small system of such equations is:

$$\begin{aligned} x &= \{0, x, y\} \\ y &= \{\{x\}\} \end{aligned}$$

We can give a precise formulation of the relevant equations as follows: By analogy with the expansion from $\mathbb{R}$ to $\mathbb{C}$, we need to introduce indeterminate elements. To take the step from $\mathbb{R}$ to $\mathbb{C}$, we first need to obtain the domain $\mathbb{R}[x]$ of polynomials in the indeterminate x; then $x^2 + 1 \in \mathbb{R}[x]$ is an expression for which we seek a root; moreover, any root will be an expression that does not implicitly contain the indeterminate x. Likewise, we will expand V with a *class* X of indeterminates, one for each set in V: $X = \{x_a : a \in V\}$.[7] And the "polynomials" we obtain—which we will call *complex sets*—are sets built up from other sets together with elements of X. As a simple example, consider the complex set $A(x_a, x_b) = \{0, \{1, x_a\}, (x_b, 2)\}$, where $x_a, x_b \in X$. One of the equations that we wish to be able to solve is

$$x_a = A(x_a, x_b).$$

A solution to such an equation will be a set whose build-up does not contain any of the elements of X; such a set is called a *pure set*.

The *Solution Lemma*, which is equivalent in ZFC^- to AFA, gives a precise statement of classes of equations that we wish to consider, and asserts that any such system of equations always has a unique solution.

Theorem 5. (Solution Lemma) *Suppose A_x is a complex set, for each $x \in X$. Then the system of equations*

$$(**) \qquad x = A_x$$

has a unique solution; that is, there is a unique family $\{b_x \mid x \in X\}$ of pure sets such that for each $x \in X$, and each indeterminate x_a occurring in A_x, if we replace in A_x each such occurrence of x_a with b_{x_a}, and if we denote the resulting set B_x, then, for each $x \in X$,

$$b_x = B_x.$$

Therefore, a ZFC^- + AFA universe is obtained as a universe that includes the well-founded sets and provides unique solutions to all equations of the form $(**)$.

We now return to our discussion of Ω as a model of pure consciousness. We consider next how all sets can be seen to arise from and return to Ω, in such a way that unity dominates.

Ω As the Only Reality

So far in this article we have seen how Ω in a ZFC^- + AFA universe, as a model of pure consciousness, captures the dynamics of pure consciousness in ways that the empty set, in a ZFC universe, cannot. We have seen that dynamics of Ω originate with its "collapse" to a point—indicated by the fact that it satisfies the equation $x = \{x\}$—paralleling the Vedantic perspective, elaborated by Maharishi, that the dynamics of unfoldment of consciousness within itself, into the Veda and the universe, begin with the collapse of "A" to "K," of unboundedness of *Atma* to a point within *Atma*. Although the empty set also exhibits the dynamics of collapse from $\emptyset$ to $\{\emptyset\}$, in this case, this "collapse" results in a separation of the expressed value from its source—we have $\emptyset \neq \{\emptyset\}$ in contrast to $\Omega = \{\Omega\}$—and in that sense represents the sprouting instead of *pragya-aparadh*. We have also seen how the appearance of three from one, displayed in the dynamics of pure consciousness as

[7] More formally, we are re-building the universe starting with atoms or *urelements* at the 0th stage. Urelements are sets, different from the empty set, that have no elements. See [2] for a full treatment.

the emergence of Rishi, Devata, and Chhandas, has a parallel in the dynamics of Ω, as evidenced by the facts that, respectively, $\Omega = \Omega^{\Omega}$, $\Omega = \Omega : \Omega \to \Omega$, and $\Omega = \{\Omega\}$.

The Vedantic insight we wish to explore in this final section is the view that everything is "nothing but" pure consciousness; that everything is nothing but the dynamics of pure consciousness. The reality is one; differences arise as a point of view, a way of looking at or conceiving, this one reality. The reality of the all-pervasiveness of pure consciousness is expressed this way in the *Yoga Vasistha* [16]:

> What appears as the world to the conditioned mind is seen by the unconditioned mind as Brahman. (p. 506)

Also:

> When pure consciousness alone exists, pervading everything inside and out, how does the notion of division arise, and where? (p. 513)

And the answer to this rhetorical question given in Maharishi's Vedic commentaries [8] is that division arises by virtue of the principle of *vivarta*; that is, division is only an appearance:

> Here, unity [in the Samhita of Rishi, Devata, and Chhandas] appears to be diversity (Rishi, Devata, and Chhandas). This is the absolute eternal principle of *vivarta*, where something appears as something else. (p. 589)

Once again, using the empty set within a ZFC universe as a model of pure consciousness falls short as we seek to model the "nothing but pure consciousness" principle. Even though the empty set is at the core of every set, differences among sets in the universe dominate. The entire enterprise of modern mathematics relies on the fact that sets that do not have precisely the same elements are *different sets*. In no mathematical sense can it be said, for example, that $\{1, 2\}$ and $\{0, 4, 9\}$ are "the same." The fact that these sets have a common source in the empty set is a *hidden reality*, not a "living" reality.

The reason that this fundamental unity is not seen "on the surface" of mathematics is, we suggest, because of the fact that even the dynamics of $\emptyset$ are hidden from view, in contrast with Ω whose internal dynamics are seen explicitly in its representation as a graph. In fact, as we observed at the end of the last section, the difference between the empty set and Ω is captured nicely in contrasting their respective apgs:

In the single-loop graph, we see a picture of a self-relationship, an inherent dynamism between the vertex and itself. One way to view the single-loop graph that pictures Ω is as a kind of *refinement* of the single-vertex graph that pictures $\emptyset$, in the sense that the self-referral dynamics that one may imagine are "hidden" deep within the empty set have been brought into plain view (Figure 13). The edge from the single vertex to itself that is added to the single-vertex graph can be seen as symbolic of "reconnecting" the point to itself.

This viewpoint provides a new way of viewing all sets in the standard ZFC universe V. In V, all sets are seen as distinct and unrelated, even though the "core" of every set is always simply the empty set. We can use our insights about the relationship between $\emptyset$ and Ω to explicitly *reconnect* every set to its "source" in the following way.

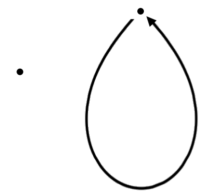

Figure 13. Contrasting the Graphs of $\emptyset$ and Ω

First, let us observe that every well-founded set can be pictured by an apg that has exactly one childless vertex, and in every case, this childless vertex is decorated with the empty set. This is true because, for every set A, by the Axiom of Foundation (as we have observed), every maximal $\in$-chain starting at A is finite and terminates in $\emptyset$; in particular, there is a natural number n and sets $x_0, x_1, \ldots, x_n$ such that:

$$\emptyset = x_0 \in x_1 \in \ldots \in x_{n-1} \in x_n = A.$$

Therefore, as in the different apg pictures of the set $\{0, 1\}$ discussed earlier (shown below in Figure 14), all edges pointing to $\emptyset$ can be directed to a single vertex decorated with $\emptyset$, and this vertex is necessarily childless (since $\emptyset$ has no elements). We shall call any such picture of a well-founded set a *canonical picture* of the set.

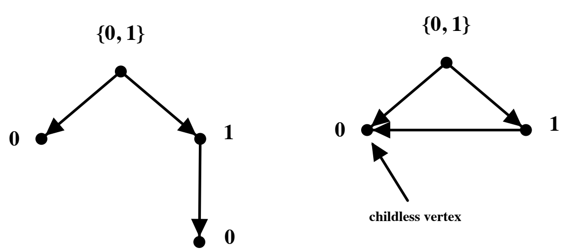

Figure 14. Canonical and Non-canonical Pictures of the Same Set

Then, to reconnect any well-founded set A to its source, we can simply add one edge to a canonical picture from the vertex decorated with $\emptyset$ to the designated point, decorated with A; that is (if for the moment we name vertices by their decorations), we add to the graph the edge $\emptyset \to A$. We will call the edge that is added in this way the *reconnecting edge*; notice that there is, for any canonical picture of a well-founded set, *just one* reconnecting edge. Here is an example:

In Figure 15, we begin with a canonical picture of the well-founded set $\{0, 1\}$; recall that $0 = \emptyset$ and $1 = \{\emptyset\}$, so the graph on the left is also a canonical picture of $\{\emptyset, \{\emptyset\}\}$. Its unique childless vertex is located in the lower left of the picture, labeled by $0 = \emptyset$. The middle graph shows what happens when we reconnect this vertex to its source by adding an edge from 0 to $\{0, 1\}$. The decoration will necessarily change because of the addition of the reconnecting edge (so no decoration is shown in the middle graph). Finally, in the third apg, we attempt to decorate the graph

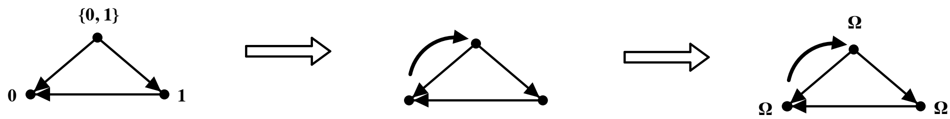

Figure 15. Adding a Reconnecting Edge to a Canonical apg

in the middle with some set. Certainly Ω can be used, as one may easily verify. But now because every apg has a *unique* decoration, the *only* way to decorate this middle graph is by placing Ω at every vertex.

What the example shows is that, by adding the reconnecting edge to a canonical apg of a well-founded set, everything about the set, including its elements and internal relationships, reveals itself to be nothing but Ω.

We can look at this example in a somewhat different way. As we observed above, the graph on the left in Figure 15 pictures the set $\{\emptyset, \{\emptyset\}\}$. When we add the reconnecting edge to the apg, the effect is the same as substituting Ω for each occurrence of $\emptyset$, and so the apg on the right in Figure 15 is in fact $\{\Omega, \{\Omega\}\}$, which can easily be seen to equal Ω. The point here is that the set obtained by adding the reconnecting edge is built up in exactly the same way from Ω as the original set was built from $\emptyset$. But in the Ω case, though the dynamics are the same, all that is actually ever built in the process is Ω.

This example provides a good analogy for the Vedantic insight that all the transformational dynamics of pure consciousness are self-referral dynamics in which pure consciousness remains pure consciousness, as described in the *Yoga Vasistha* [16]:

> Thus the pure consciousness brings into being this diversity with all its names and forms, without ever abandoning its indivisibility.... (p. 638)

In our example, it was clear that adding the reconnecting edge produces an apg that pictures Ω. In that example, one notices that after adding the reconnecting edge, every vertex has a child. In fact this is always what happens: Whenever we start with a canonical apg and join the only childless vertex to the distinguished point, the result is an apg in which every vertex has a child. By the following Theorem, it follows that adding the reconnecting edge always produces an apg that pictures Ω.

Theorem 6. (The Ω-Theorem) *An apg is a picture of Ω if and only if every vertex of the apg has a child.*

By the Ω-Theorem, the "fundamental reality" underlying each well-founded set can be discovered by adding a single reconnecting edge to its canonical picture, from the vertex labeled with 0 (or $\emptyset$) to the designated point of the apg.

This insight about the structure of sets captures in a mathematical way the dawning of the vision of Vedanta, in which every object is recognized to be nothing but pure consciousness.

Even certain experiential aspects of this awakening are modeled here: In Maharishi's [7] treatment, the full awakening to Brahman occurs first in the blossoming of experience and then is completed with one final stroke of knowledge. That final stroke of knowledge comes from the imparting of a *mahavakya* when the student's experience is "ripe." Well-known examples of mahavakyas from the Vedic literature include *tat tvam asi* (That thou art)[8] and *sarvam khalvidam brahma* (all this is Brahman).[9] Maharishi explains [7]:

> Brahman becomes an all-time reality through the mahavakyas. See, through the experience everything is recognized in terms of the Self, but that experience in terms of the Self becomes significant through the teaching, because through the teaching it comes onto the level of understanding. Experience is one thing, understanding is another, and only when it comes onto the level of understanding does it become established everywhere. Then its all-pervadingness becomes a living reality. (p. 316)

Elaborating further, he explains:

> But when the experience is *ripe* and the teacher says "tat tvam asi—really you are That," it's a *revelation.* He may have known "tat tvam asi" before, but that "tat tvam asi" did not pinpoint that experience. (p. 318)

In our model based on Ω, we see that the "awakening" to the reality that every set is nothing but Ω arises from a "final stroke," represented by a single reconnecting edge. This one small change in the viewpoint regarding any given set reveals that the set's true nature, including the very way it is built up from its origin, is nothing but Ω.

We make one further observation about our model: One can apply the procedure of introducing reconnecting edges to every stage V_α of the universe. In Figure 16, this is illustrated with a canonical apg for V_3 and the result of adding the reconnecting edge. The example shows that the "reality" of V_3, accessed when the reconnecting edge is introduced is, as described above, simply Ω. Likewise, each stage in the construction of the universe V is transformed into Ω by introducing a reconnecting edge. Forming the union of all these refined stages, as one does to build V, results in a union of many copies of Ω; in the end we just end up with Ω.

The same thing happens if we picture the universe V itself with one enormous canonical apg: When we introduce just one reconnecting edge, the entire universe is seen to be nothing other than Ω, and yet the dynamics of set formation can still be seen in the apg after that edge is inserted. Here, with one "final stroke of knowledge," the detailed inner dynamics of Ω are fully displayed, and yet all transformations are seen to be nothing other than dynamics of Ω. This example provides a parallel for the reality embodied in the mahavakya *sarvam khalvidam brahma* (all this is Brahman).

We have used our examples in Figures 15 and 16 to illustrate how Ω can be appreciated as the pervasive reality of any well-founded set. These examples also show how all well-founded sets *originate from* Ω. This becomes apparent as we observe in these examples that by *removing* the reconnecting edge, the original apg, and hence the original set, comes back into view. For example, removing the reconnecting edge in Figure 15, Ω is transformed back to the set $\{0, 1\}$.

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[8] *Chhandogya Upanishad,* 6.11

[9] *Chhandogya Upanishad,* 3.14.1

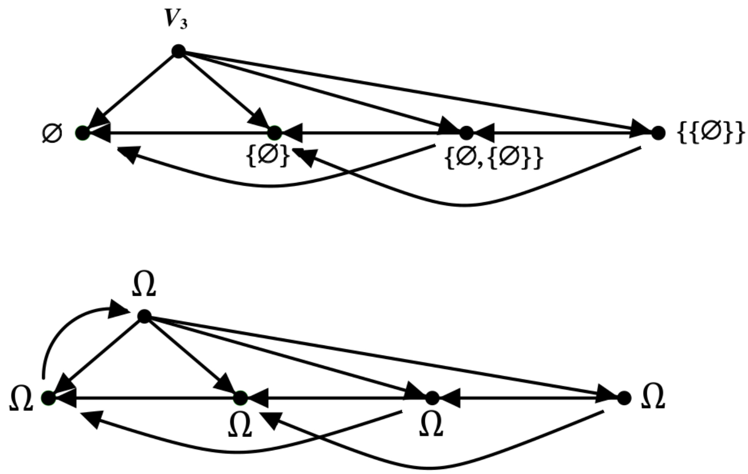

FIGURE 16. V_3 Before and After Adding the Reconnecting Edge

Generalizing a bit, let us define U to be the set of all apgs (G, p) that picture Ω and in which there is an edge $e : u \to p$, removing which produces a well-founded apg with point p. All elements of the vast collection U are pictures of Ω. At the same time, one can say that every well-founded set "arises from" the act of removing a single edge from some apg in U, and thereby breaking the apg's (and the set's) connection to its source.

As we discussed in the first section, from the Vedantic perspective, the viewpoint that takes an expressed value to be disconnected from its source is the nature of *pragya-aparadh*, and arises because of the principle of *vivarta*. We see the mechanics of the emergence of *pragya-aparadh* in this mathematical model. First, when the perfectly balanced state in which the large apg for V, plus reconnecting edge, which pictures Ω and all its internal dynamics, undergoes the *transition* to the apg for V *without* its reconnecting edge, we see the actuality of diversified values of sets. This is precisely the nature of *vivarta* according to Maharishi [8]: "The actuality of *vivarta* is realized in the transition state, where the self-referral dynamics of *Atyantabhava* (absolute abstraction), without losing its self-referral status, appears to become *Anyonyabhava*" (p. 589). Elaborating further: "Here, Unity (Samhita) appears to be diversity (Rishi, Devata, Chhandas)" (p. 589).

At the same time, the diversity that emerges as the universe V results in a loss of unity; the connection of each set in V to its source (accessible by introducing a reconnecting edge) is obscured.

5. CONCLUSION

With the aim of clarifying the vision of Vedanta, we have sought in this article a foundations-based mathematical model that could give adequate expression to the internal dynamics of pure consciousness and the relationship of those dynamics to the manifest field of existence, the universe itself. We showed that the standard

ZFC universe provides a reasonable analogy for manifest existence and the empty set naturally plays the role of pure consciousness. Because the empty set is, like pure consciousness itself, devoid of individual content, and because it is (as we showed) at the core of every individual in the universe, we suggested that, at least in these respects, the mathematical universe together with the empty set provides a reasonable model for manifest existence and its relationship to pure consciousness.

On closer examination, though, we found that the empty set fails to exhibit characteristics and dynamics of pure consciousness that are key elements in the Vedantic vision. These elements can be summarized as follows:

(1) *Rishi, Devata, Chhandas.* Pure consciousness, being conscious, automatically assumes the roles of knower, known, and process of knowing, unfolding a three-in-one structure within itself

(2) *Akshara: Collapse of infinity to a point.* In the process of knowing itself, it locates a point within itself; the dynamics that follow, indicated by a verse within the Rik Veda itself, involve a "collapse" of unboundedness to the point. From this collapse arises a sequential unfoldment of the structuring impulses of the Veda and *Vishwa*. In these dynamics, unboundedness and point are nothing other than pure consciousness in different modes.

(3) *Unity Consciousness.* The reality of the manifest universe is that it is nothing other than the internal dynamics of pure consciousness; a material universe is only appearance, whose reality is pure consciousness alone. The appearance of pure consciousness as the universe is due to the principle of *vivarta*. The state of consciousness that takes this appearance to be separate from, distinct from, pure consciousness is *pragya-aparadh*, the mistake of the intellect. Observed differences and distinctions are, when seen from the vision of unity, "transparent"; what dominates the enlightened vision is the unity among all objects of perception, and connectedness to their source as pure consciousness.

We discovered that if there could exist some set x that satisfies the equation $x = \{x\}$, such a set could be a good model for (1). This is because, as we showed, once $x = \{x\}$ is known to hold, it also follows that $x = x^x$ as well, and from these we can conclude that x is Rishi, being equal to the totality of all its transformations (namely, x^x); x is Devata, since $x \in x^x$, whence $x : x \to x$; and x is Chhandas, being equal to its own objectification as $\{x\}$. At the same time, being just x in every case, it is the samhita (unity) of Rishi, Devata, and Chhandas.

For (2), we have seen that the transformational dynamics, which begin with the emergence of three from one, arise from the very existence of an x for which $x = \{x\}$. This equation indicates, as was mentioned, that x is equal to its own objectification as a point. From this relationship emerges the division into three, and, as we argued before, the further structuring impulses of knowledge and natural law, as embodied in the *evaluation map*: $x = x : x^x \times x \to x$. Such an x plays the role of each structuring impulse of nature, remaining all the while as nothing but x itself.

We observed that, using the usual ZFC axioms for set theory, there can be no such set x since such a set cannot be well-founded. We suggested considering an expansion of the standard well-founded universe V by adjoining to V the ideal element Ω, which does satisfy the equation $x = \{x\}$ (together with many other such non-well-founded elements). We accomplished this step by replacing the usual ZFC Axiom of Foundation with the Anti-Foundation Axiom, or AFA. Any universe of $ZFC^- + AFA$ is, essentially, an expansion of the usual well-founded universe to include Ω and other ideal elements. Indeed, the well-founded part of any such expansion is isomorphic to the well-founded universe we started with.

The AFA guarantees that any graph (apg) has a unique decoration; Ω is the unique decoration for the single-loop graph (Figure 7). This picture of Ω gives a visual illustration of its own self-referral dynamics; it is because of these dynamics that Ω can provide a model of (1) and (2).

For (3), working in a $ZFC^- + AFA$ universe, we showed how every well-founded set's essential nature as Ω can be unveiled by introducing a single reconnecting edge to a canonical picture of the set. The reconnecting edge serves to "awaken" every vertex of the apg to its underlying reality as Ω. The unique decoration for this reconnected apg is a graph each of whose vertices is decorated with Ω itself, including the distinguished point. Adding the reconnecting edge is analogous to the delivery from the master of a *mahavakya*, providing the final impulse of knowledge—for the student who is "ripe" for enlightenment—needed to realize the final truth that "all this is Brahman." Conversely, we saw that removing the reconnecting edge from any such apg has the effect of restoring the original set, with all its distinctions within itself and from other sets. In this way, well-founded sets were seen to "emerge from" Ω; each emerges from Ω by removing the reconnecting edge of a suitable picture of Ω. The underlying truth of this "manifested" set is seen in introducing the reconnecting edge once again; this step reveals in the resulting apg the dynamic relationships within the set as a variation of Ω alone. Viewing the manifested set instead as truly disconnected from its source—with the reconnecting edge removed—illustrates the state of *pragya-aparadh*, where differences dominate and individuals appear to be cut off from their source.

This mathematical context—a model $\hat{V}$ of $ZFC^- + AFA$—gives us, therefore, two views of the unfoldment of the universe of mathematics. One view is the usual one, in which sets unfold stage by stage from the empty set, each set being different from every other and for the most part disconnected from any kind of source. This view arises from cutting away all ideal elements of $\hat{V}$, leaving only disconnected well-founded sets. The other view is a world of sets that is identical, set for set, to the well-founded universe V itself, except that the distinctions between sets have become transparent; what dominates in this second view is the reality of every set as Ω. This view arises when each well-founded set's canonical picture is reconnected to itself, connecting its unique empty set vertex to itself. Transition from the first to the second view parallels awakening to Brahman, wherein every individual, and the universe V itself, are seen to be nothing but Ω itself. The transition from the second view to the first parallels the emergence of *pragya-aparadh*, creating the appearance of separation of sets from their source, arising from cutting away the connection of each set with itself.

This mathematical vision of creation as being the dynamics of pure consciousness alone illumines the eternal truth from the *Yoga Vasistha* [16]:

> The ignorant regard this samsara as real. In reality it does not exist at all. What does exist is in fact the truth. But it has no name! (p. 528)

6. Appendix: The Bisimulation and Bisimilarity Relations

Earlier in this paper, we gave an overview of the construction of a model $\hat{V}$ of ZFC^- + AFA. The idea we discussed was that $\hat{V}$ should consist of equivalence classes of apgs, where two apgs are to be considered equivalent if they picture the same set. Two such apgs are said to be *bisimilar.* We pointed out that, for formal correctness, the bisimilarity equivalence relation must not actually be defined in terms of sets that have not yet been constructed. In this Appendix, we describe the right way to define bisimilarity. The first step is to define the *bisimulation* relation on graphs, and then to define bisimilarity as a special kind of bisimulation.

Suppose $G = (M_G, E_G), H = (M_H, E_H)$ are graphs. A *bisimulation* for G, H is any relation $R \subseteq M_G \times M_H$ for which there is another relation R^+, with $R \subseteq R^+ \subseteq M_G \times M_H$, having the property that, for each $a \in M_G$ and $b \in M_H$, aR^+b if and only if both of the following hold:

(i) whenever $a \to x$ there is $y \in M_H$ with $b \to y$ and xRy
(ii) whenever $b \to y$ there is $x \in M_G$ with $a \to x$ and xRy.

The equivalence relation that we will need is a *maximum* bisimulation; we will discuss this concept after giving an example.

Example 1. Consider the two apgs mentioned earlier that picture the set $\{0, 1\}$; for this example, we will call them G and H; see Figure 17. We have labeled

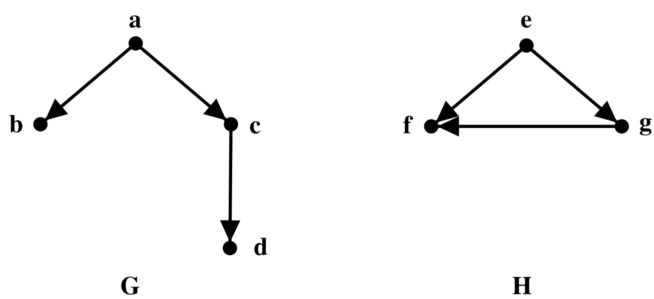

Figure 17. Bisimulations and Bisimilar Graphs

these graphs differently here to emphasize the fact that bisimulation relations make sense for any kind of directed graph, not just apgs. We define three bisimulations for G, H. The first is a trivial bisimulation, which is obtained by declaring that childless vertices are related to each other:

$$R_1 = \{(b, f), (d, f)\}.$$

The conditions for a bisimulation are satisfied vacuously because none of the vertices in the relation have children.

The second bisimulation includes vertices that do have children:

$$R_2 = \{(c, g), (d, f), (b, f)\}.$$

The new pair (c, g) can be included because the respective children of c and g occur as pairs also. Notice that $R_2' = \{(c, f), (d, f), (b, f)\}$ is *not* a bisimulation since, although $c \to d$, there is no d' in H such that $f \to d'$. Also, $R_2'' = \{(c, g), (b, f)\}$ is not a bisimulation because, though $g \to f$ in H, there is no corresponding child of c that is paired with f in the relation.

The third bisimulation includes all the vertices.

$$R_3 = \{(a, e), (c, g), (d, f), (b, f)\}.$$

It is not hard to see that there is only one bisimulation that includes all the vertices; it is called the *total* bisimulation, or the *maximum* bisimulation. Two graphs that admit such a bisimulation are called *bisimilar*. Considering these graphs as apgs (with respective designated points a and e), it is apparent in this example that, although the graphs are not isomorphic, the membership structures that they specify are the same; it is clear in this case that these apgs must picture the same set. In this example, we write $G \equiv H$ to indicate that the graphs are bisimilar. □

Bisimilarity can be shown to be an equivalence relation on directed graphs. Moreover, the discussion in the example gives some idea about why this equivalence relation is the one we are seeking: When we consider apgs as displays of potential membership structure, it seems intuitively clear that whenever apgs are bisimilar, the membership structure of both apgs is the same, so they should picture the same set.

Using this equivalence relation $\equiv$ on apgs, we wish to form the collection of resulting equivalence classes. Since typically each equivalence class will itself be a proper class, we use a familiar technique (known as *Scott's trick*) to reduce their size: we take a representative r from each such class having least rank and we let $[r]$ denote the set of all apgs equivalent to r and having the same rank as r; and finally, we let $\hat{V}$ denote the collection of all such reduced equivalence classes $[r]$.

We have described the "sets" of our new universe. We also need to specify the "membership relation." Suppose $[r], [s] \in \hat{V}$. Since r, s are pointed graphs, we may write $r = (G, p_G)$ and $s = (H, p_H)$, where p_G and p_H are the designated points of the apgs. We declare that $[r]$ is a "member of" $[s]$ if there is a vertex q in H with $p_H \to q$ such that the sub-apg Hq of H determined by q (defined as: $Hq = \{h \in M_H \mid \text{there is a path in } H \text{ from } q \text{ to } h\}$) is bisimilar to G:

$$\exists q \in M_H \; Hq \equiv G.$$

We give a simple example:

Example 2. Figure 18 displays two apgs G and K. We indicate here why the equivalence class $[G]$ is an "element" of $[K]$, according to our new definition. We show that G is equivalent to a sub-apg of K whose designated point is a child of u. Here, there is only one way for this to happen since u has only one child, namely, e. Clearly, the sub-apg of K whose designated point is e is precisely the apg H of Example 1, which, as we have seen, is indeed equivalent to G. Therefore, $[G]$ is an "element" of $[K]$. Of course, this is what we expect since G is a picture of $\{0, 1\}$ and K is a picture of $\{\{0, 1\}\}$. □

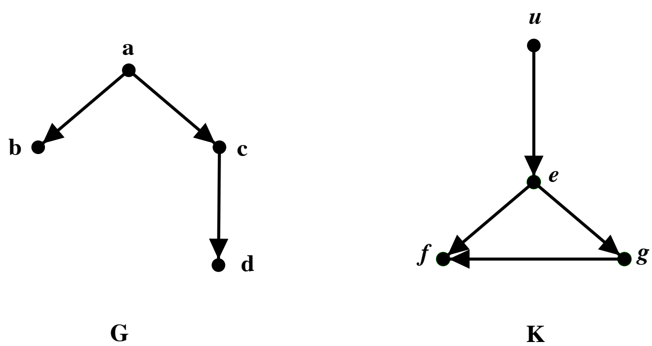

FIGURE 18. The Membership Relation Between Equivalence Classes of Bisimilar Graphs

It can be shown that the equivalence classes belonging to $\hat{V}$ that contain well-founded trees correspond exactly to the sets in the original universe V; more precisely, if we let $\mathrm{WF} = \{[r] \in \hat{V} \mid r \text{ is well-founded}\}$, then $V \cong \mathrm{WF}$.

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International Journal of Mathematics and Consciousness

In recent centuries, scientists have found that many phenomena in nature obey physical laws that can be expressed precisely in the language of mathematics. Their successes have led scientific inquiry beyond the physical world to include what was previously considered metaphysical or non-material. Today, an increasing number of scientists are examining the nature of consciousness and its relationship to the human brain.

While most models of consciousness propose that it is a product of chemical and electrical behavior within the brain, no current theory resolves the so-called "hard problem of consciousness"—how physical processes in the nervous system give rise to subjective experiences such as experiencing, thinking, feeling, analyzing, and creating. At the same time, it is undeniable that without awareness—without consciousness—we cannot think, perceive, dream, or love. On this basis alone, a scientific journal dedicated to exploring the nature of consciousness is timely and appropriate.

While consciousness can be studied within a variety of disciplines, mathematics especially lends itself to examine the relationship between consciousness and physical phenomena. Mathematics is precise and rigorous in its methodology, giving symbolic expression to abstract patterns and relationships. Although developed subjectively, using intuition along with the intellect and logical reasoning, mathematics allows us to make sense of our outer physical universe. Mathematics is the most scientific representation of subjective human intelligence and thought, formalizing how individual human awareness perceives, discriminates, organizes, and expresses itself.

The scientific consideration of consciousness by itself is a formidable challenge, for consciousness is a purely abstract reality. But the study of what we might call "consciousness at work"— how consciousness expresses itself in our daily activity of thinking, analyzing, creating, theorizing, and feeling—is inherently more accessible. For this exploration also, mathematics is the ideal tool, because its ability to express patterns of abstract human awareness helps us make sense of our physical universe. One could in fact argue that mathematics is the most scientifically reliable tool for the exploration of the dynamics of consciousness, for it alone can be seen as the symbolic representation of "consciousness at work."

The International Journal of Mathematics and Consciousness will help to fulfill the need for a forum of research and discussion of consciousness and its expressions. The editors invite mathematicians, scientists, and other thinkers to present their theories of consciousness without restriction to proposed axioms and postulates, with the stipulation only that such theories follow strict logical argumentation and respect proven facts and observations. Articles that use factual or logical counterarguments to challenge commonly believed but not fully established facts and observations are also welcome.