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EDITORIAL INTRODUCTION TO THE JOURNAL

Throughout history, natural phenomena have been ultimately mysterious. Some of these phenomena were explained by religious belief, others by philosophical analysis. Since the 17th century, the modern scientific approach has found that many phenomena in nature obey clearly describable physical laws. This success greatly widened the ambit of scientific inquiry beyond the physical into the realm of what previously had been considered metaphysical or nonmaterial. Today, the territory of scientific inquiry has expanded to include how matter leads to consciousness.

Most common and popular models of consciousness share the postulate that physical activity in the brain is prior to consciousness. No current theory, however, has been able to resolve the problem of how physical processes in the brain give rise to subjective experiences. Even quantum mechanical theories, while suggesting potential mechanisms that might create "unexplainable" phenomena, fall short of answering the fundamental questions about subjective experience. This gap—between the objective, material brain and the intimately known, private qualia of subjective experience, or "what it is like" to experience something—has so far not been bridged. Some thinkers have even rejected qualia out of hand, asserting that we have insufficient knowledge of the physical world to evaluate their existence.

Some believe that early *Homo sapiens* depended entirely on sensory experience as a reference for what does and does not exist, and that only as our understanding evolved did we come to challenge the evidence of our senses. Certainly, the discoveries of modern science changed the way we looked at the world. They gave us intellectual models of the universe that often seemed to contradict our sensory model but which provided in fact more accurate pictures and were eventually confirmed by experimental observation.

Perhaps the most notable example is the shift from a geocentric to a heliocentric view of the cosmos as a result of the work of Copernicus, Kepler, and Galileo in the 16th and 17th centuries. More recently, inquiry into very small and very large time and distance scales in relativity theory, quantum mechanics, quantum field theory, and cosmology has radically changed our beliefs about the nature of matter and physical phenomena as our senses perceive and our intellects apprehend them. We may ask, what actually exists for us? And we may agree that everything is continuously changing; we may even agree that whatever appears not to change is only one of an infinite number of simultaneously existing possibilities. For example, in some models a particle can be everywhere at once, and the fact that we find it here and now suggests either that we have collapsed the infinitude of its possibilities in a single act of conscious experience or that it continues to exist everywhere in an infinite number of universes parallel to the one in which we experience it.

In all this uncertainty, one fact seems undeniable: the fact of our own awareness. Without awareness, we can neither perceive nor apprehend, neither see nor think nor dream. Commonly, this awareness is called consciousness: the observer, the

witness, the experiencer. If indeed this is the one undeniable fact, then it is timely that a scientific journal be dedicated to the study of consciousness as primary.

To be truly scientific requires that the journal obey rigorous methods of logic, research, and experimentation. At the same time, this requires that no *a priori* or unproven points of view stand in the way of new original postulates, previously explored theories revisited with new insights, or unconventional axioms.

The *International Journal of Mathematics and Consciousness* is founded in part to fulfill this need. The *Journal* opens the door to all mathematicians, scientists, and thinkers to present their theories of consciousness and the consequences thereof. With the requirement that such theories follow strict mathematical, logical argumentation and respect proven facts and observations, articles can be submitted for review, without restriction to their proposed axioms and postulates. The *Journal* also welcomes carefully reasoned articles that challenge commonly held, but not fully established, theories and beliefs.

1. Consciousness and "Consciousness at work"

Abstract concepts and subjective experiences such as love, friendship, beauty, devotion, happiness, inspiration, pain, despair, and deception, are, in and by themselves, hard to study scientifically because of their innate, subjective, personal nature. Even more difficult to study is the more abstract consciousness that seems to be like a screen on which these emotions, notions, and sensations are projected and experienced.

Modern cognitive neuroscience identifies various neural correlates of these mental states. The discipline of psychology attracted great thinkers who proposed various theories and methods of investigation, mostly focusing on the manifestations, observable or subjectively reportable signs and symptoms, and causes and effects of such inner experiences. Physicists recently have attempted to bridge the gap between the physical world and conscious experience through various quantum mechanical models.

Philosophy, metaphysics, and spiritual and religious studies delve into ontological, epistemological, and other fundamental questions, using more or less formal logic or a wide variety of opinions and postulates. In contrast, art forms such as music, painting, and fictional writing are outer expressions of inner experiences and creative thinking.

All theories, concepts, and creative work, whether scientific, psychological, philosophical, artistic, or spiritual are the manifestations of "consciousness at work." While it might be challenging to study "consciousness" as such, in and by itself, it may be easier to study "consciousness at work"—its dynamics and its manifestations.

The postulates that can be made about consciousness as an abstract phenomenon or epiphenomenon are most amenable to investigation by scientifically analyzing and studying "consciousness at work." The *International Journal of Mathematics and Consciousness* invites analyses of consciousness at work from various perspectives with a particular emphasis on mathematics.

2. Mathematics

Mathematics studies abstract forms, patterns, relationships, and transformations in an exact, systematic, and logical way. Forms and shapes like circles and triangles are the subject of geometry and topology. Patterns of number and operations lead to algebra. Relationships that change in time form the basis of calculus. Mathematics also includes the study of mathematics itself. The study of mathematical reasoning is undertaken by logic. Even questions about the limits of the mathematical method and the nature of mathematical knowledge can be addressed using the methodology of mathematics.

Using mathematical models of experimental observations of the physical world makes it possible to give a purely abstract formulation of real-life phenomena. Subjective mathematical reasoning, which is nevertheless entirely rigorous, applied to these models leads to new descriptions and predictions about the world.

Mathematics is fundamentally a method that finds patterns of orderliness in the subjective field of human intelligence and thought. Based on sets of axioms and postulates that are accepted without proof, mathematics gives a structure to the way our minds and intellects operate. It systematizes how individual human awareness perceives, discriminates, organizes, and expresses its own patterns of functioning. In our opinion, mathematics is certainly one of the most useful and scientifically manageable methods to study the interface between consciousness and physical phenomena.

Mathematics is in essence a subjective discipline that nevertheless allows us to organize and make sense of the physical universe in which we exist. Though subjective, it is precise and effective in objective scientific explorations. It is a fundamental and indispensable tool of all sciences, and at the same time, it is an expression of abstract human awareness and intellect.

3. Mathematics and Consciousness

The *International Journal of Mathematics and Consciousness* takes the position that methods of mathematics and mathematical modeling provide especially appropriate tools for studying the interface between consciousness and physical phenomena. As we have pointed out above, mathematics is a fundamental and indispensable tool of all sciences, and at the same time an expression of abstract human awareness and intellect. It is therefore the most precise scientifically reliable tool in the exploration of the dynamics of consciousness. It can be seen as the precise abstract representation of consciousness at work.

The ways in which human beings explore and express the experience of consciousness are as varied as nature itself. The following list contains some of the relevant sciences and other forms of human inquiry:

(1) Physics and chemistry (physical/quantum mechanical theories of consciousness at work)
(2) Biology and cognitive neuroscience (biological/electro-chemical/neural correlates of consciousness at work)
(3) Mathematics (abstract representation of consciousness at work)

(4) Psychology and cognitive sciences (objectification of subjective experiences of consciousness at work)
(5) Economics, particularly behavioral economics (production, distribution, and consumption of resources as models of the dynamics of consciousness at work)
(6) Philosophy (discursive representation of consciousness at work)
(7) Arts (subjective creative representation of consciousness at work)
(8) Religion (individual/group belief in the origins and dynamics of consciousness and consciousness at work)
(9) Spirituality (personal and totally subjective experience of consciousness at work)
(10) Study of pure consciousness itself (the field or screen "phenomenon" on which or by which all aspects of consciousness at work take place)

The *International Journal of Mathematics and Consciousness* maintains the position that of all such pursuits, mathematics, because of its rigor, depth, and effectiveness, is the most suitable discipline to study the interface between consciousness and the physical world. This *Journal* is devoted to exploring this interface using the rigorous approach of mathematics. We invite all mathematicians, scientists, and thinkers to submit papers using a mathematical approach to consciousness and "consciousness at work" in all its aspects.

Tony Nader, MD, PhD, M.A.R.R.

GEODESICS IN MIND AND MATTER UNDER A PRINCIPLE OF LEAST EFFORT

RIAZ AHMED

Abstract. General relativity tells us that, in the absence of non-gravitational forces, matter follows the path of a geodesic, which is the shortest natural path through spacetime. We investigate whether something similar happens in the mind during the practice of the Transcendental Meditation technique, a simple mental technique that allows the mind to effortlessly transcend to Transcendental Consciousness, the most expanded level of the mind. We use the principle from general relativity that describes the motion of matter and light in spacetime in the absence of external forces, namely that light bends near massive bodies and traces the geodesic path determined by the curvature of spacetime. Using the principle of least action, we will show how the trajectories of mental activity can be compared to physical geodesics in spacetime. We will compare the natural path of matter or energy in spacetime to the effortless flow of the mind during the practice of the Transcendental Meditation technique. We propose that, with regular practice of the Transcendental Meditation technique, one's everyday mental activity follows natural paths that expand the conscious capacity of mind and develop the full potential of the individual. Overall, we offer a comparative analysis in which both mind and matter move along intrinsic, geodesic-like natural paths under appropriate conditions.

1. Introduction

In General Relativity (GR), the motion of a free particle has a precise meaning if no non-gravitational forces act; in this case, the particle follows a geodesic, a path determined entirely by the curvature of the spacetime, and any departure from that path signals the presence of forcing. We use this same principle to frame a description of the natural tendency of the mind in the practice of the Transcendental Meditation® program, in which the mind transcends to Transcendental Consciousness by eliminating external forcing factors with the help of a mantra. We model mental activity as a trajectory in a space of mental states equipped with a cost structure that represents effort. In this setting, a "least-effort" trajectory plays the role of a mental geodesic, while stress and strain control appear as forcing terms that bend the trajectory away from its natural path. The point is not to identify mind with spacetime, but to use a familiar extremal-path framework to clarify what "effortless" could mean and what kinds of patterns it might predict.

In GR, matter is described by fields defined on spacetime, including densities, pressures, and velocities for a fluid, or other matter fields depending on the model. The physical state of the matter during any event is simply the collection of values

Key words and phrases. General Relativity; Geodesic motion; Vedic Science; Transcendental Meditation; Consciousness.

these fields take there, and their dynamical evolution is given by a system of partial differential equations, coupled to Einstein's field equations. For a body made of such matter, this system typically has an initial-value formulation, meaning that once suitable initial field data are specified, the future behavior of the body is determined in principle by the equations. What is often most useful, however, is not every internal detail of that evolution but the motion of the body "as a whole." In the regime where non-gravitational forces are negligible, GR predicts that this overall motion is governed by the geodesic principle: the representative worldline of a freely falling body follows a timelike geodesic. This idea has two sides at once. It is geometric, because a geodesic is defined by the Lorentzian metric through its Levi–Civita connection, and it is variational, because the same curve arises by extremizing the proper-time functional [1, 2, 3]. This combination of geometry and extremal principles provides a natural template for thinking about other systems in which one distinguishes between natural motion and motion driven by forcing, including the dynamics of attention. This dual viewpoint is attractive when we look at another domain where people regularly speak about what is "natural" and what is "forced," namely the inner life of attention. We shall see that the laws governing motion in both instances are quite similar.

In much the same spirit, Maharishi Mahesh Yogi presents the Transcendental Meditation technique as a simple response to the external factors of stress and strain that does not rely on concentration or forceful control of attention; instead, the mind is gently allowed to settle inward, step by step, drawn by its natural tendency to move toward greater happiness. This technique is described as a simple, natural, effortless procedure in which the mind settles inward, at times transcending to Transcendental Consciousness, a state characterized as restful alertness. Research on the Transcendental Meditation technique, conducted in more than 250 universities and research institutions [4, 5] over the past 45 years, has shown that practitioners experience a high level of brain integration both during and after the practice of the Transcendental Meditation technique.

In this paper, we propose a simple bridge between two ideas, one that belongs to modern science and and one that belongs to Vedic science, based on the principle of the science of consciousness that order is present everywhere, governed by universal natural laws. The remainder of the paper reviews the geometric background, develops the state-space model, and discusses implications within the context of GR and Vedic Science.

2. A geometric model for motion of matter

General relativity changed the way we understand gravity. Gravity is not a force, but the geometry of spacetime, and that geometry determines how matter moves. The basic geometric concept is the *metric* g, which is a rule for measuring intervals. In local coordinates $(x^\mu) = (t, x, y, z)$, the metric allows one to compute the spacetime interval

$$ds^2 = g_{\mu\nu}(x)\, dx^\mu dx^\nu. \tag{2.1}$$

We can think of a geodesic as the path that goes as straight as possible without leaving the spacetime surface, just as great circles play that role on a sphere

and straight lines on a flat plane. In GR the same idea holds in four-dimensional spacetime: a geodesic is the straightest possible worldline according to the metric g. A worldline is a geodesic if it has *zero curvature with respect to the spacetime geometry*. In coordinates, this becomes the differential equation

$$\frac{d^2x^\mu}{d\tau^2} + \Gamma^\mu_{\alpha\beta}(x)\,\frac{dx^\alpha}{d\tau}\frac{dx^\beta}{d\tau} = 0. \tag{2.2}$$

The coefficients $\Gamma^\mu_{\alpha\beta}$ are Christoffel symbols built from the metric that tells you how curved the coordinates are in spacetime. The Christoffel symbols are determined entirely by g and their first derivatives, and they vanish in flat spacetime in standard inertial coordinates. The key physical message is simple: *free fall means geodesic motion*. If no non-gravitational forces act, then the object follows equation (2.2). When extra influences are present, the equation gains a forcing term on the right hand side:

$$\frac{d^2x^\mu}{d\tau^2} + \Gamma^\mu_{\alpha\beta}\,\frac{dx^\alpha}{d\tau}\frac{dx^\beta}{d\tau} = \frac{1}{m}F^\mu, \tag{2.3}$$

where F^μ represents the applied four-force and m the mass. Thus GR cleanly separates two ingredients: the *intrinsic* motion set by geometry (geodesics) and *deviations* produced by forcing.

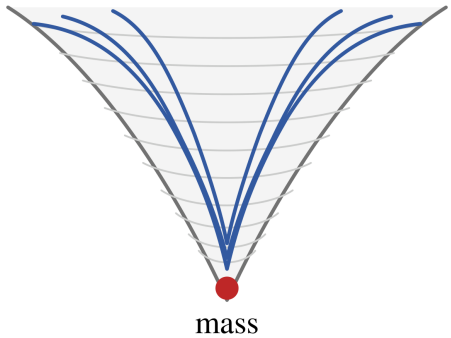

FIGURE 1. This figure shows that spacetime curvature guides freely falling matter along geodesics that bend along the natural path set by the curvature of the geometry.

3. A STATE-SPACE MODEL FOR MENTAL MOTION

The GR discussion above suggests a useful template: when forcing is small, motion is largely determined by the intrinsic structure of the space. When forcing is larger, trajectories are deflected. To apply this idea to mental activity, we need only a modest mathematical representation of "mental motion." We do *not* assume that the mind is spacetime but that the flow of thoughts and way of transcending our mind to a state of Transcendental Consciousness is natural. We only borrow the same style of description: a state, a rule for measuring change, and a way to represent forcing.

Let M denote a *state space* of mental activity. A point $m \in M$ represents the mind at a moment, summarized by a few coordinates. For example, one may take

$$m(t) = \big(r(t),\, b(t),\, s(t)\big), \tag{3.1}$$

where r is respiration rate, b is behavioral steadiness or EEG coherence measures, and s represents stress. As time passes, the mental episode traces a curve $m(t)$ through M in the same way a worldline $x^\mu(\tau)$ traces motion through spacetime.

To speak meaningfully about "effort" or "strain," we introduce a simple cost for change. The most conservative choice is a weighted quadratic cost on the rate of change,

$$\mathcal{E}[m] = \int_{t_1}^{t_2} \frac{1}{2} \sum_i w_i\, \dot{m}_i(t)^2\, dt, \tag{3.2}$$

with weights $w_i > 0$ that set how costly it is to change each coordinate. This already captures an important idea: rapid, sudden shifts are more costly than smooth motion. In geometric terms, equation (3.2) corresponds to choosing a constant "metric" $G_{ij} = \mathrm{diag}(w_1, w_2, w_3)$ so that the local cost of a small change is

$$ds^2 = G_{ij}\, dm^i dm^j. \tag{3.3}$$

One can allow G_{ij} to depend on m if the cost of change depends on the current state; for example, changes in attention may be harder when stress is high, but the constant case is enough to motivate the framework. When the cost metric depends on m, the same minimization yields the geodesic equation on (M, G),

$$\ddot{m}^k + \widetilde{\Gamma}^k{}_{ij}(m)\, \dot{m}^i \dot{m}^j = 0, \tag{3.4}$$

where $\widetilde{\Gamma}^k{}_{ij}$ are the Christoffel symbols computed from G_{ij}. This is directly parallel to equation (2.2). It is a statement about intrinsic motion defined by a chosen geometry of cost.

To better capture real-life conditions, we include a forcing term. Stressful sensory load, deliberate control, and environmental factors can all be represented as this forcing input.

$$\ddot{m}^k + \widetilde{\Gamma}^k{}_{ij}(m)\, \dot{m}^i \dot{m}^j = \mathcal{F}^k(t). \tag{3.5}$$

Equation (3.5) is the direct counterpart of the forced geodesic equation (2.3). In this model, an "effortless" process corresponds to a regime where $\mathcal{F}(t)$ is small and the trajectory remains smooth and low-cost with respect to G. This is the sense in which the Transcendental Meditation technique is described as allowing attention to settle inward rather than pushing it. This is not a field-theory model of mind, it is a language choice that helps distinguish intrinsic flow from forced deviation. Figure 2 compares weak and strong forcing in the (state 1, state 2) plane. With small $\mathcal{F}$, the path is only slightly changed and still settles into the same spiral, but with large $\mathcal{F}$, the input pushes the system and drives it away along a clear direction.

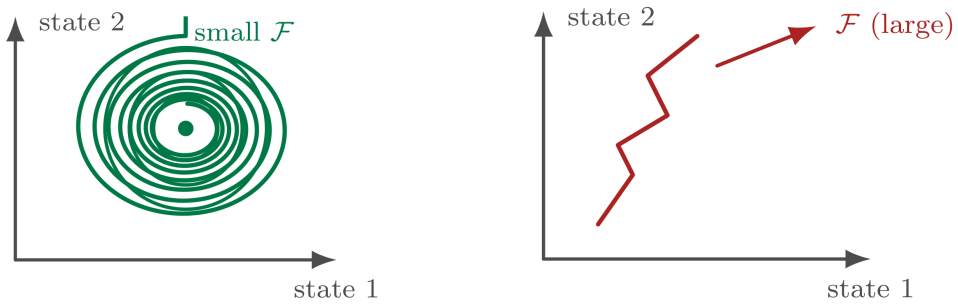

FIGURE 2. When $\mathcal{F}$ is small, it only gives a gentle push and the trajectory still spirals toward a stable point. When $\mathcal{F}$ is large, the push is strong and the trajectory moves across the plane instead of returning to the usual spiral behavior.

The Transcendental Meditation program as a reduced-forcing technique. The reason I borrow the geodesic idea from GR is that it gives a clean way to talk about motion that happens "on its own" when forcing is reduced. This approach is also reflected in Consciousness-Based[SM] education, where the focus is not only on what is learned but on developing the learner's inner capacity and clarity [6]. In spacetime, a freely-falling body is not being constantly steered; its geometry is already defined by spacetime, and the geodesic equation simply records that the path is determined intrinsically by the metric rather than by external pushes.

Maharishi also describes the Transcendental Meditation procedure in simple terms as effortless—something that does not require controlling the mind, but allowing it to settle naturally when strain is not added [7]. The most relevant concept already exists in ancient Vedic literature where the Transcendental Meditation technique is a way of transcending the mind to Transcendental Consciousness without any external control just as matter takes the natural path when there are no external forces acting on it. In Consciousness-Based education, this same emphasis on developing the inner basis of experience appears in more academic form; for example, Gorini notes that practical techniques including the Transcendental Meditation and TM-Sidhi program are used for developing consciousness in a systematic way in the classroom [8].

4. Discussion

A guiding principle in the science of consciousness is that *order is present everywhere*—in nature, in physiology, and in the dynamics of the mind—and that this order becomes clearer when disturbance and strain are reduced. Order exists throughout life in nature, in the body, and in the mind, and this order becomes easier to see when disturbance and strain are reduced. This idea connects naturally to the main theme of our paper. However, GR gives a precise picture of how order can arise. Gravity is not treated as an extra push from the outside, it is built into the geometry of spacetime. When an object is not acted on by other forces, it does

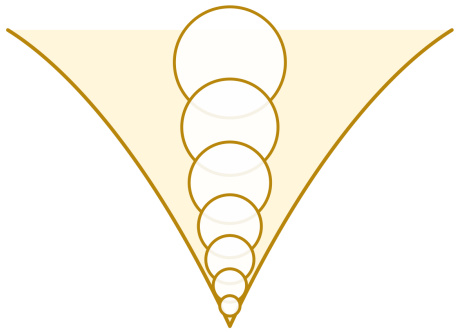

FIGURE 3. Reduced forcing allows mental activity to settle inward along a natural path toward a deeper, more stable level of experience during the practice of the Transcendental Meditation® technique.

not need constant correction. It simply follows a geodesic, the natural path set by the spacetime metric. In that setting, the most "ordered" motion is the simplest one, and any departure from it points to additional influences. Similarly, when the mind is not influenced by external forces, it follow a natural path toward its least excited state, Transcendental Consciousness.

In this paper, we use the geodesic idea as a careful mathematical way to express a point about mental functions. As forcing is reduced, trajectories should become more direct, smoother, and require less control. The geodesic study proposed here is not meant to reduce or replace the richness of lived experience. However, meditators experience that the effortless practice of the Transcendental Meditation technique is smoother and easier.

Also, we experience that reducing external forcing reveals a more natural order in both matter and mind, linking the geodesic idea in GR with the Vedic view of settled awareness, using the Transcendental Meditation technique as a tested technique to examine this effect.

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THE SŪRYĀ-VIVĀHA SŪKTA AND THE BṚHADRATHA TRADITION: A *NAKṢATRA*-BASED RECONSTRUCTION OF THE VERNAL EQUINOX IN THE SAVITĀ–ARJUNĪ–MAGHĀ REGION, C. 35,700 BCE

RUPA BHATY AND UMESH KUMAR SINGH, PHD

Abstract. This paper examines the astronomical, ritual, and textual evidence for locating the Ṛgvedic memory of Sūryā's bridal hymn and King Bṛhadratha in the Maghā epoch of approximately 34,500 BCE. The study begins with RV 10.85, the Sūryā-vivāha sūkta, where the back-to-back *nakṣatra*s Aghā/Maghā and Arjunī/Phalgunī are mentioned in connection with Sūryā's bridal procession. The hymn states that Agni led the procession, the Aśvins acted as groomsmen, the oxen were driven in Aghā/Maghā, and Sūryā was borne in Arjunī. When examined alongside the marriage rule of Āśvalāyana Gṛhya Sūtra, which prescribes marriage during the northern course of the Sun, the waxing fortnight, and an auspicious *nakṣatra*, the hymn preserves an ancient memory of marriage aligned with the vernal equinoctial transition into Arjunī/Phalgunī.

The paper further integrates evidence from the Maitrāyaṇīya Āraṇyaka/ Upaniṣad, where Bṛhadratha of the Ikṣvāku lineage is associated with cosmological observations, including the movement of the pole star, changes in stellar longitudes, and the decline of ocean levels. The references to Bṛhadratha in Ṛgveda I.36.18 and X.49.6 are connected with the Bṛhadratha of the Maitrāyaṇīya tradition, indicating a shared memory of an ancient royal observer. The *Maghādi* system in the Maitrāyaṇīya text, together with the Agni–*vasanta* association and the Śraviṣṭhārdha reference, is interpreted as preserving the astronomical position of the vernal equinox in the Maghā–Phalgunī region.

Using *nakṣatra* divisions, precessional calculation, and Stellarium-based testing, the study identifies Regulus, the *yogatārā* of Maghā, as a discrete marker at the boundary of Maghā and Arjunī around 34,500 BCE. The correction of the later *Maghādi* date by one precessional cycle places the event between approximately 33,880 and 34,780 BCE, closely matching the proposed epoch. The convergence of Ṛgvedic, Gṛhya Sūtra, Maitrāyaṇīya, *nakṣatra*, precessional, and geological evidence supports the conclusion that the Sūryā bridal hymn and Bṛhadratha tradition preserve a precise astronomical memory of the vernal equinox in the Savitā–Arjunī–Maghā region during the 35,700 BCE epoch.

1991 *Mathematics Subject Classification.* 01A32, 85-03.

Key words and phrases. Ṛgveda, Sūryā-vivāha, Bṛhadratha, Maghā, Arjunī, Phalgunī, Regulus, *nakṣatra*, precession of the equinoxes, Maitrāyaṇīya Upaniṣad, archaeoastronomy.

1. Introduction

The present paper develops a consolidated study of two related arguments: the *nakṣatra*-based astronomical reading of Sūryā's bridal hymn in RV 10.85, and the identification of King Bṛhadratha with a Maghā epoch of approximately 34,500 BCE. The first argument rests upon the occurrence of Aghā/Maghā and Arjunī/Phalgunī in the Sūryā-vivāha hymn and the climatic conditions in Ṛgveda. The second rests upon the Maitrāyaṇīya Āraṇyaka/Upaniṣad (hereafter MAU), where Bṛhadratha of the Ikṣvāku lineage is associated with ancient cosmological observation, including the movement of Dhruva (the pole star), changes in stellar longitude, and the decline of ocean level and other geo-hazards. [1, 2].

These two arguments converge upon a single astronomical thesis: the vernal equinoctial point was observed in the Maghā–Arjunī region, with Regulus functioning as a discrete marker at the boundary of Maghā and Arjunī. The ritual language of Sūryā's procession, the Gṛhya Sūtra rule prescribing marriage during the northern course of the Sun, and the Maitrāyaṇīya reference to *Maghādi* are treated here as complementary witnesses to the same inherited memory.

The goal of this paper is not to detach these materials from their ritual and symbolic forms, but to show how ritual symbolism may encode a *nakṣatra*-based astronomical framework. In this framework Agni marks the vernal equinoctial point, Sūryā moves into the domain of Soma, the Aśvins appear as celestial groomsmen, and Bṛhadratha preserves a royal observational memory transmitted, in the MAU, across an earlier textual layer.

2. Sources and Method

The primary Vedic datum for this paper is RV 10.85, the Sūryā-vivāha sūkta; the key mantra is RV 10.85.13, in which Aghā and Arjunī occur in immediate sequence. The ritual datum is drawn from the Āśvalāyana Gṛhya Sūtra, which preserves the rule of marriage during *udagayana*, the waxing fortnight, and an auspicious *nakṣatra* [8]. The supporting cosmological and royal datum is drawn from the Maitrāyaṇīya Āraṇyaka/Upaniṣad, especially its references to Bṛhadratha, *Maghādi*, Agni, *vasanta*, and Śraviṣṭhārdha [5, 10].

The method is synthetic. First, the *nakṣatra* indications in RV 10.85 are examined against the translation of H. H. Wilson, which follows the traditional interpretation of Sāyaṇa [11]. Second, the marriage rule of Āśvalāyana is read as a possible ritual continuation of the same memory. Third, the MAU Bṛhadratha material is used to identify the broader epoch. Finally, precessional calculation and the position of Regulus are applied to locate the proposed event around 34,500 BCE, using the twenty-nine-*nakṣatra* scheme (Maitrāyaṇī Saṃhitā, 29th nakṣatra, brāhmaṇa nakṣatra somo devatā)[1] in which the equinoctial point's passage through one *nakṣatra* division corresponds to approximately 888 years (see §11 for the corrected precessional constant).

[1]Rupa Bhaty, "Were There Thirty-Four *Nakṣatras* in Remote Antiquity? A Vedic and Mathematical Inquiry" (paper presented at the WAVES Conference, Maharishi International University, USA, July 31–August 2, 2026).

The following verse occurs in the *Ṛgveda Vedāṅga Jyotiṣa* (RVJ 14) in connection with the algorithm for determining the prevailing *nakṣatra* at any given *parvan* (lunar fortnight or syzygy):

> jau drā gaḥ khe śve 'hir ro ṣā cin mū ṣaṇ ṇyaḥ ṣū mā dhā ṇaḥ|
> re mṛ ghā svā po jaḥ kṛ ṣyo ha jye ṣṭhā ity ṛkṣa-liṅgaiḥ|| — *RVJ* 14

In this verse, the 27 *nakṣatras* are encoded mnemonically using single syllables or short phonemes. These symbols represent either the initial or final syllables of the *nakṣatra* names, or, in several instances, the names of their presiding deities:

No.	Syllable	Referent / Deity	Corresponding *Nakṣatra*
1	*jau*	*Āśvayujau*	Aśvinī
2	*drā*	*Ārdrā*	Ārdrā
3	*gaḥ*	*Bhagaḥ* (Deity)	Pūrvaphalgunī
4	*khe*	*Viśākhe*	Viśākhā
5	*śve*	*Viśvedevāḥ* (Deity)	Uttarāṣāḍhā
6	*hir*	*Ahirbudhnyaḥ* (Deity)	Uttaraproṣṭhapadā
7	*ro*	*Rohiṇī*	Rohiṇī
8	*ṣā*	*Āśleṣā* (*Āśreṣā*)	Āśleṣā
9	*cit*	*Citrā*	Citrā
10	*mū*	*Mūlā*	Mūlā
11	*ṣaṇ*	*Śatabhiṣaj*	Śatabhiṣaj
12	*ṇyaḥ*	*Bharaṇyaḥ*	Bharaṇī
13	*ṣū*	*Punarvasū*	Punarvasu
14	*mā*	*Aryamā* (Deity)	Uttaraphalgunī
15	*dhāḥ*	*Anurādhāḥ*	Anurādhā
16	*ṇaḥ*	*Śravaṇaḥ*	Śravaṇa
17	*re*	*Revatī*	Revatī
18	*mṛ*	*Mṛgaśīrṣa*	Mṛgaśīrṣa
19	*ghāḥ*	*Maghāḥ*	Maghā
20	*svā*	*Svātī*	Svātī
21	*paḥ*	*Āpaḥ* (Deity)	Pūrvāṣāḍhā
22	*jaḥ*	*Aja-Ekapāt* (Deity)	Pūrvaproṣṭhapadā
23	*kṛ*	*Kṛttikāḥ*	Kṛttikā
24	*ṣyaḥ*	*Puṣyaḥ*	Puṣya
25	*ha*	*Hastaḥ*	Hasta
26	*jye*	*Jyeṣṭhā*	Jyeṣṭhā
27	*ṣṭhāḥ*	*Śraviṣṭhā*	Śraviṣṭhā

The sequence begins with *Aśvinī* (*Āśvayujau*) and proceeds by selecting every sixth *nakṣatra* from the canonical list. Notably, several *nakṣatras* in this schema (specifically items 3, 5, 6, 14, 21, and 22) are denoted by the names of their presiding deities rather than the names of the *nakṣatras* themselves.[2] This type of schema, in which both the names of the Nakṣatras and sometimes presiding deities occur together, was also prevalent in the Ṛgveda Saṃhitā (See Section 3, Sūryā's Bridal Hymn) and appears to preserve some of the latest memories recorded in the Ṛgveda.

[2]B. N. Narahari Achar, "Searching for nakṣatras in the Ṛgveda," *Electronic Journal of Vedic Studies* 6, no. 2 (2000), doi:10.11588/ejvs.2000.2.1522.

TABLE 1. The 28 *nakṣatras* with presiding deities, Kaye's star identifications, sidereal longitude, and latitude according to the *Sūrya-Siddhānta*.

No.	*Nakṣatras*	Deity (TS/RVJ)	Kaye	SS Long. Sidereal Yogatārā	SS Lat.
1	Aśvinī	Aśvin	β, γ Arietis	8°	10°
2	Bharaṇī	Yama	35, 39, 41 Arietis	20°	12°
3	Kṛttikā	Agni	η Tauri, etc.	$37^\circ 30'$	5°
4	Rohiṇī	Prajāpati	$\alpha, \theta, \gamma, \delta, \epsilon$ Tauri	$49^\circ 30'$	-5°
5	Mṛgaśīrṣa	Soma	λ, ϕ_1, ϕ_2 Orionis	63°	-10°
6	Ārdrā	Rudra	α Orionis	$67^\circ 20'$	-9°
7	Punarvasu	Aditi	β, α Geminorum	93°	6°
8	Puṣya	Bṛhaspati	θ, δ, γ Cancri	106°	0°
9	Āśleṣā	Sarpa	$\epsilon, \delta, \sigma, \eta, \rho$ Hydrae	109°	-7°
10	Maghā	Pitṛ	$\alpha, \eta, \gamma, \zeta, \mu, \epsilon$ Leonis	129°	0°
11	Pūrvā Phālgunī	Aryamā	δ, θ Leonis	144°	12°
12	Uttarā Phālgunī	Bhaga	β, 93 Leonis	155°	13°
13	Hasta	Savitā	$\delta, \gamma, \epsilon, \alpha, \beta$ Corvi	170°	-11°
14	Citrā (mid point)	Indra	α Virginis	180°	-2°
15	Svātī	Vāyu	α Bootis	199°	37°
16	Viśākhā	Indrāgnī	$\gamma, \beta, \alpha, \iota$ Librae	213°	$-1^\circ 30'$
17	Anurādhā	Mitra	δ, β, π Scorpii	224°	-3°
18	Jyeṣṭhā/ Rohiṇī	Indra	α, σ, τ Scorpii	229°	-4°
19	Mūla/ Vicṛtau	Pitṛ	$\lambda, \mu, \kappa, \iota, \theta, \epsilon$ Scorpii	241°	-9°
20	Pūrva Āṣāḍhā	Āpaḥ	δ, ϵ Sagittarii	254°	$-5^\circ 30'$
21	Uttara Āṣāḍhā	Viśvedevā	$6, \zeta$ Sagittarii	260°	-5°
22	Abhijit	Brahmā	α, ϵ, ζ Lyrae	$266^\circ 40'$	60°
23	Śravaṇa	Viṣṇu	α, β, γ Aquilae	280°	30°
24	Śraviṣṭhā	Vasu	γ-Delphini	290°	36°
25	Śatabhiṣaj	Indra	λ Aquarii, etc.	320°	$-0^\circ 30'$
26	Pūrva Bhādrapada	Aja-ekapāt	α, β Pegasi	326°	24°
27	Uttara Bhādrapada	Ahirbudhnya	γ Pegasi, α Andro.	337°	26°
28	Revatī	Pūṣā	γ Piscium, etc.	$359^\circ 50'$	0°

The fourth column of Table 1, headed *Kaye*, records the identification of each nakṣatra with specific stars or star groups following the scheme of G. R. Kaye. These identifications have been adopted here as reproduced in Mahavir Prasad Srivastava's *Vijñāna Bhāṣya* edition of the *Sūrya-Siddhānta*, where Kaye's correspondences are tabulated alongside the traditional data.[3]

The list of *Nakṣatras* and their corresponding deities (whose associations became more elaborate in the later Brāhmaṇa texts and are discussed in detail in my other paper) is presented here to familiarize the reader with the system of celestial observation encoded in the Ṛgvedic account of Sūryā's Bridal Procession.

[3]Mahavir Prasad Srivastava. *Sūrya-Siddhānta (Vijñāna Bhāṣya)*. Allahabad: Dr. Ratnakumari Swadhyay Sansthan, 1987 (Reprint of 2nd ed., 1982; 1st ed. 1940), pp. 626–627.

3. Sūryā's Bridal Procession in RV 10.85

RV 10.85 describes the celestial marriage of Sūryā. The hymn presents Sūryā's journey to her husband's house through a sequence of ritual, cosmic, and astronomical images. Agni leads the procession, the Aśvins are the attendants, and Sūryā is conveyed through the *nakṣatra* field. The following verses form the immediate background:

> स्तोमा॑ आसन्प्रति॒धय॑: कु॒रीर॑ं छन्द॑ ओप॒श:।
> सू॒र्यायां॑ अ॒श्विना॑ व॒राग्निरा॑सीत्पुरोग॒व:॥ RV 10.85.8[4]
>
> "Hymns were the cross-bars (of the car); the Kurīra metre was the thong of the whip; the Aśvins were Sūryā's groomsmen; Agni was the leader of the procession."
>
> सोमो॑ वधू॒युर॑भवद॒श्विना॑स्तामु॒भा व॒रा।
> सू॒र्यां॑ यत्पत्ये॒ शंस॑न्ती॒ं मनसा॑ सवि॒तादद॑दात्॥ RV 10.85.9
>
> "Soma was desirous of a bride; the two Aśvins were the two groomsmen when Savitā gave Sūryā, who was ripe for a husband, [to Soma endowed] with intelligence."

The first of these verses states that the Aśvins were the bridegroom's companions and that Agni led the procession. The second states that Soma desired the bride and that Savitā possibly refering to Hasta gave Sūryā to her husband. The below *mantra* explicitly shows Soma as a distinct *nakṣatra* deity.

> स॒जूर्मि॒त्रावरु॑णाभ्यां स॒जू: सोमे॑न॒ विष्णु॑ना ।
> आ यां॑ह्यग्ने अत्रि॒वत्सु॒ते र॑ण ॥ RV 5.51.9
>
> Accompanied by Mitra and Varuṇa, accompanied by Soma and Viṣṇu, come, Agni and like Atri, delight in the libation.

The hymn thereby presents a celestial marriage in which Agni, Soma, Savitā, the Aśvins, and Sūryā function as an organized astronomical procession rather than as isolated mythic figures.

The crucial mantra is RV 10.85.13:

> सू॒र्यायां॑ व॒हतु॒: प्रागा॑त्सवि॒ता यम॒वासृ॑जत्।
> अ॒घास्॑ु हन्यन्ते॒ गावोऽर्जु॑न्योॅ: पर्यु॑ह्यते॥
>
> "Sūryā's bridal procession, which Savitā despatched, has advanced; the oxen are driven along in the Aghā (constellations); she is borne [to her husband's house] in the Arjunī (constellations)."

This verse gives the *nakṣatra* sequence on which the argument rests. The cattle are driven in Aghā/Maghā and Sūryā is borne into Arjunī/Phalgunī. Since Aghā and Arjunī are contiguous *nakṣatra*s, the verse is read here as preserving a transitional and precessional moment: the bride advances from the Hasta (Deity: Savitā) side and is borne into Arjunī, which in the present reading corresponds to the vernal equinoctial transition into Arjunī/Phalgunī. It should be noted that this

[4]Commentary by Sāyaṇa (Ṛgveda-bhāṣya): the "thong of the whip" (or tuft of hair; cf. *go opaśa*, RV 6.53.9); the Aśvins were Sūryā's groomsmen; Prajāpati Savitā is said to have offered his daughter Sūryā to Soma, cf. Aitareya Brāhmaṇa 4.7. Translation follows [11], which is based on Sāyaṇa's commentary.

astronomical reading of RV 10.85.13 is one interpretive layer among several possible readings of the verse; the majority of Vedic commentators and translators, including Sāyaṇa and Wilson, treat the verse primarily as calendrical-ritual imagery for the timing of a wedding procession rather than as a fixed-point stellar record, and this more conservative reading is noted here for balance.

4. Agni as the Leader and the Vernal Point

The description of Agni as *purogava*, the leader of the procession, is central to the astronomical reading proposed here. In the ritual and calendrical symbolism of Vedic literature, Agni is repeatedly associated with beginnings, with the east, with *vasanta* (spring), and with the forward movement of the year. The Sūryā hymn places Agni at the front of the bridal procession; read together with the MAU evidence discussed below, Agni is here interpreted as a marker of the vernal equinoctial point.

On this reading, Sūryā, the solar bride, approaches the equinoctial point; Agni leads the transition; the Aśvins appear as groomsmen; in the opposite sky in the west, and in the east, the movement from Hasta/Savitā into Arjunī/Phalgunī observation encodes the moment at which the vernal equinox lay at the boundary of these two *nakṣatra* regions.

The Gṛhyasūtras attest that marriages, Vasantotsava, Madanotsava, and various other rituals were traditionally performed during the spring season when the Sun starts rising and setting in the Northern Hemisphere.

5. The Marriage Rule of the Āśvalāyana Gṛhya Sūtra

The ritual counterpart of the Ṛgvedic image is found in the Āśvalāyana Gṛhya Sūtra:

> उदगयन आपूर्यमाणपक्षे कल्याणे नक्षत्रे चौलकर्मोपनयनगोदानिववाहाः।[5]

The rule prescribes that marriage, along with rites such as *cūḍākarman*, *upanayana*, and *godāna*, should be performed during the northern course of the Sun (*udagayana*), in the waxing fortnight, and under an auspicious *nakṣatra* [8]. This is read here as a possible ritual continuation of the older memory contained in the Sūryā-vivāha sūkta, though the Sūtra text itself is many centuries later than the Rigveda and does not itself specify which *nakṣatra* corresponds to *udagayana* in any given era.

A later rule permits marriage at all times:

> सार्वकालमेके विवाहम्।[6]

This later relaxation is consistent with, though it does not prove, the possibility that the older rule once held more specific seasonal significance tied to the position of the equinox, which – owing to precession – would have drifted through the *nakṣatra*s over the centuries and eventually made a fixed *nakṣatra*-based rule impractical.

[5] Āśvalāyana Gṛhya Sūtra 1.4.1
[6] Ibid. 1.4.2

6. Aryaman, Arjunī, and Conjugal Prosperity

The *nakṣatra* Pūrva Phalgunī is associated in later astrological tradition with Aryaman, a deity of marital bond and social order, which is offered here as corroborating, though not decisive, support for reading Arjunī/Phalgunī as a *nakṣatra* of marriage. Aryaman's connection with marriage also appears in RV 5.3.2:

> त्वमर्यमा भवसि यत्कनीनां नाम स्वधावन्गुह्यं बिभर्षि।
> अञ्जन्ति मित्रं सुधितं न गोभिर्यद्दम्पती समनसा कृणोषि॥
>
> "You are Aryaman in relation to maidens; you bear a mysterious name, O self-sustaining one; they anoint you, as a welcome friend is anointed with milk and butter, when you make husband and wife of one mind."

This mantra speaks of Aryaman in relation to maidens and to the making of husband and wife of one mind, which is consistent with – without independently proving – the proposed marital significance of the Sūryā hymn's movement into Arjunī/Phalgunī.

7. The Climatic Conditions during Ṛgvedic Times

The expression *śatāhimāḥ* (शतहिमाः) in Ṛgveda 6.12.6 appears to preserve a memory of colder climatic conditions in the Ṛgvedic period. The Ṛgveda mentions the seasons of spring (*vasanta*), summer (*grīṣma*), and autumn (*śarad*). It is also familiar with the rainy season (*prāvṛṣ*) and winter (*hemanta/himā*), but it does not mention *śiśira* as a distinct season. This seasonal classification may preserve a memory of climatic conditions characteristic of the Late Pleistocene.

The refrain "शतहिमाः सुवीराः" occurs at the conclusion of seven hymns, from Ṛgveda 6.4 to 6.24. The Sixth *Maṇḍala* belongs to the Bhāradvāja family of seers, and these *śatāhimāḥ* verses are primarily addressed to Agni and Indra.

Further, Ṛgveda 6.4.8 is composed in the *Paṅkti* (पङ्क्तिः) metre and is associated with *Haimantī*, as also indicated in Yajurveda 13.54–58. This association may preserve an early ritual memory connected with the longer *hemanta* (snowy winter) season. Let us now examine the remaining evidence from the Vedic tradition to determine whether any references can be associated with the vernal equinox occurring near the *Nakṣatra* Aghā (Maghā/Arjunī).

8. Bṛhadratha in the Ṛgveda and the Maitrāyaṇīya Tradition

The second body of evidence concerns Bṛhadratha. RV I.36.18 reads:

> अग्निना तुर्वशं यदुं परावत उग्रादेवं हवामहे।
> अग्निर्नयन्नववास्त्वं बृहद्रथं तुर्वीतिं दस्यवे सहः॥
>
> "We invoke, from afar, along with Agni, Turvaśa, Yadu, and Ugradeva; may Agni, the arrester of the robber, bring hither Navavāstva, Bṛhadratha, and Turvīti."

RV X.49.6 also mentions Bṛhadratha:

> अहं स यो नववास्त्वं बृहद्रथं सं वृत्रेव दासं वृत्रहारुजम्।
> यद्वर्धयन्तं प्रथयन्तमानुषग्दूरे पारे रजसो रोचनाकरम्॥

> "I, the slayer of Vṛtra, am he who crushed Navavāstva and Bṛhadratha, [who crushed] Dāsa as [I crushed] Vṛtra, when I drove, one after the other, to the distant shore of the shining world, both my rising and my far-spreading [foes]."

The identification of the Ṛgvedic Bṛhadratha, a defeated chieftain associated with the Dāsa/Dasyu side in these two verses, with the Bṛhadratha of the Maitrāyaṇīya tradition – there remembered instead as an Ikṣvāku king in a mood of world-weary renunciation – is the single most speculative step in the argument, since the name Bṛhadratha is not uncommon in the epic and Purāṇic king-lists and could in principle refer to different figures of different periods. The paper's working hypothesis is that these are traditions about one composite or paradigmatic figure; this is stated here explicitly as an assumption rather than an established identity, pending onomastic and genealogical study beyond the scope of the present paper.

The Maitrāyaṇīya tradition remembers Bṛhadratha as an Ikṣvāku king: बृहद्रथेक्ष्वाकुवंशध्वजशीर्षात्मजः। as in the fuller passage: अथ भगवाञ्छाकायन्यः सुप्रीतोऽब्रवीद्राजानं महाराज बृहद्रथेक्ष्वाकुवंशध्वजशीर्षात्मजः कृतकृत्यस्त्वं मरुन्नाम्नो विश्रुतोऽसीत्ययं वाव खल्वात्मा ते कतमो भगवान्वर्ण्य इति तं होवाचेति ॥ 2।1 ॥[7]

9. The Maitrāyaṇīya *Maghādi* System

The MAU material is read here as supplying the broader astronomical frame. The *Maghādi* sequence is interpreted as marking the beginning of the Agni (solar/fiery) half of the year; since Agni is elsewhere connected with *vasanta*, the *Maghādi* reference is read as an indication that the vernal equinox stood, at some point, in the Maghā region. The Śraviṣṭhārdha (Dhaniṣṭhā-half) reference then marks the opposite point in the cycle, corresponding to the Saumya (lunar/Soma) side of the annual round:

> अन्नं वा अस्य सर्वस्य योनिः कालश्चान्नस्य सूर्यो योनिः कालस्य तस्यैतदूपं यन्निमेषादिकालात्सम्भृतं द्वादशात्मकं वत्सरमेतस्याग्नेयमर्धमर्धं वारुणं मघाद्यं श्रविष्ठार्धमाग्नेयं क्रमेणोत्क्रमेण सार्पाद्यं श्रविष्ठार्धान्तं सौम्यम्...॥ 6.14॥
>
> "Food (*anna*), verily, is the source (*yoni*) of all this [universe]. Time (*kāla*) is the source of food. The sun (Sūrya) is the source of time. Its (time's) form is this: the year (*saṃvatsara*), accumulated from moments and so forth, consists of twelve parts. Of this, one half is *āgneya* (fiery/solar), the other half *vāruṇa* (watery/lunar): beginning with Maghā up to half of Śraviṣṭhā (Dhaniṣṭhā) is *āgneya*, in forward order; beginning with Āśleṣā (Sārpa) up to half of Śraviṣṭhā, in reverse order, is *saumya*." (Maitrāyaṇīya Upaniṣad 6.14; translation by Dr. Umesh Kumar Singh.)[8]

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[7] "Then the venerable Śākāyanya, being greatly pleased, said to the king: 'O great king Bṛhadratha, you are the head/banner of the Ikṣvāku dynasty, the son of [a worthy line]; you have accomplished your purpose (*kṛtakṛtyaḥ*), and you are renowned by the name Marut. This indeed is your Self. Which aspect of it, venerable sir, do you wish me to describe?' He (the king) said, 'Thus, please explain.'" Maitrāyaṇīya Upaniṣad 1.2–1 (numbering follows the *prapāṭhaka/khaṇḍa* division used by [5]); Sanskrit text and translation checked against the critical edition of [10].

[8] The instrumental forms *krameṇa* and *utkrameṇa* in this verse are rendered here by their ordinary lexical sense, "in [forward] order" and "in reverse order," the same sense the root

The concept of Śraviṣṭhārdha (half of Śraviṣṭhā) is mathematically viable only within a framework of either 27 or 29 *nakṣatras*. Notably, a 29-*nakṣatra* system is preserved within the Maitrāyaṇī Saṃhitā.[9]

This produces a yearly schema in which Agni is associated with the vernal side and Soma/Varuṇa with the autumnal side; the *Maitrāyaṇī Saṃhitā* additionally lists a twenty-ninth *nakṣatra* ("Brāhmaṇa *nakṣatra*/Somo devatā") with the deity Soma and the *nakṣatra* span from Maghā to Śraviṣṭhārdha marks the two halves of the year. In this schema, *Maghādi* is read not as an arbitrary list-initial but as a possible astronomical statement about the historical location of the vernal equinox at the time the underlying observational tradition originated, as distinct from the much later date at which the MAU itself was redacted.

10. Pole-Star Movement and the Decline of Ocean Level

The Bṛhadratha argument also draws on physical phenomena that the MAU associates with the king's reflections: the apparent movement of Dhruva (the pole star), changes in stellar longitude produced by precession, and a marked regression of sea level. Global sea-level reconstructions place substantial, non-monotonic falls in sea level across the last glacial cycle, with the sea level roughly 60–80 m below present around 40,000 years ago and continuing to fall, with fluctuations, toward the Last Glacial Maximum lowstand of roughly 120–130 m below present near 20,000 years ago [6]. The present paper treats the interval broadly bracketed by these reconstructions (c. 40,000–22,000 BCE) as the relevant window during which a period of comparatively rapid sea-level fall, situated by this reconstruction around 34,500 BCE, could plausibly have been the kind of abrupt environmental change that leaves a lasting imprint in oral memory, on the general principle that cultural memory more readily preserves abrupt or dramatic change than gradual change. This dating of a specific centuries-long acceleration to exactly 34,500 BCE is offered here as a working correlation rather than as a value read directly off any single

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kram carries in the standard Vedic recitational term *krama-pāṭha*, "sequential recitation" rather than as a restatement of the sun's compass direction. Some published translations, including [5], gloss this pair as the sun's "[southward]" and "[northward]" course; the square brackets in that rendering mark the solar-directional content as editorially supplied rather than a translation of *krama*/*utkrama* themselves, which denote sequence or order, not direction of motion. Grammatically, the Agni/Varuṇa (solar/lunar) division of the year is already asserted in the preceding clause (*etasyāgneyam ardham, ardhaṃ vāruṇam*); the function of *krama*/*utkrama* in the clause that follows is to specify, for each already-named half, the direction in which its two named boundary-*nakṣatras* are to be counted—a specification the sentence grammatically requires, since "from Maghā to half of Śraviṣṭhā" is directionally ambiguous on a closed circle of *nakṣatras*. That function is fully satisfied by the ordinary sense of *krama* as sequence or order; no further, unstated solar-directional sense need be supplied. The translation adopted here follows the words of the verse on this basis; text checked against [10].

[9] कृत्तिका नक्षत्रम् अग्निर् देवताग्ने रुचः स्थ प्रजापतेः सोमस्य धातुर् ऋचे त्वा रुचे त्वा भासे त्वा ज्योतिषे त्वा तेन छन्दसा तेन ब्रह्मणा तया देवतयाङ्गिरस्वद् ध्रुवा सीद रोहिणी नक्षत्रं प्रजापतिर् देवतेन्वगा नक्षत्रं मरुतो देवता बाहुर् नक्षत्रं रुद्रो देवता पुनर्वसुर् नक्षत्रम् अदितिर् देवता तिष्यो नक्षत्रं बृहस्पतिर् देवताश्लेषा नक्षत्रं सर्पा देवता मघा नक्षत्रं पितरो देवता फल्गुनीर् नक्षत्रं भगो देवता फल्गुनीर् नक्षत्रम् अर्यमा देवता हस्तो नक्षत्रं सविता देवता चित्रा नक्षत्रं त्वष्टा देवता निष्ट्यं नक्षत्रं वायुर् देवता विशाखं नक्षत्रम् इन्द्राग्नी देवतानूराधा नक्षत्रं मित्रो देवता ज्येष्ठा नक्षत्रं वरुणो देवता मूलं नक्षत्रं निर्ऋतिर् देवताषाढा नक्षत्रम् आपो देवताषाढा नक्षत्रं विश्वे देवा देवताभिजिन् नक्षत्रं ब्रह्मा देवता श्रोणा नक्षत्रं विष्णुर् देवता श्रविष्ठा नक्षत्रं वसवो देवता शतभिषं नक्षत्रम् इन्द्रो देवता प्रोष्ठपदा नक्षत्रम् अहिर् बुध्न्यो देवता प्रोष्ठपदा नक्षत्रम् अज एकपाद् देवता रेवती नक्षत्रं पूषा देवताश्वयुजौ नक्षत्रम् अश्विनौ देवता भरणीर् नक्षत्रं यमो देवता ब्राह्मणो नक्षत्रं सोमो देवताग्ने रुचः स्थ प्रजापतेः सोमस्य धातुर् ऋचे त्वा रुचे त्वा भासे त्वा ज्योतिषे त्वा तेन छन्दसा तेन ब्रह्मणा तया देवतयाङ्गिरस्वद् ध्रुवा सीद॥ मैत्रायणी संहिता 2.13.20॥

published sea-level curve, and it should be tested independently against the primary glacio-isostatic datasets (e.g. the coral-terrace and oxygen-isotope compilations underlying [6]) rather than taken as established.

The king's statement concerning the instability of the world is read here as a possible memory of astronomical and geological change rather than as a purely abstract metaphor. The Sanskrit word निमज्जनम् (*nimajjanam*, "sinking, submersion") derives from the root मज्ज् (*majj*, "to sink, plunge, immerse, dive") with the prefix नि (*ni*-, "down, into").:

> अथ किमेतैः (Now what is the use of these transitory worldly phenomena, where) वाण्र्यानां महार्णवानां शोषणं (the drying up of the vast/oceanic great oceans), शिखरिणां प्रपतनं (the falling or shattering of mountain peaks), ध्रुवस्य प्रचलनं (the displacement of the pole star from its position), व्रश्चनं वातरज्जूनां (the severing of the wind-cords that bind the heavenly bodies), निमज्जनं पृथिव्याः (the submerging or sinking of the earth in water), स्थानादपसरणं सुराणाम् (and the falling away of the gods from their celestial stations is inevitable); इत्येतद्विधे अस्मिन्संसारे (in this impermanent world of such a perishable nature,) किङ्कामोपभोगैः (what is the gain from these fleeting sensual enjoyments,) यैरेवाशितस्य (by which alone of the bound soul, who is nourished or consumed by them,) असकृदिहावर्तनं दृश्यताम् (a repeated return to this cycle of birth and death is witnessed)? इत्युद्धर्तुमर्हसि (Therefore, you are worthy to uplift and deliver me,) इत्यन्धोदपानस्थो भेक इवाहम् अस्मिन्संसारे (for I reside in this world just like a helpless frog trapped inside a blind, dark well). भगवस्त्वं नो गतिस्त्वं नो गतिः ॥4॥ (O Lord! You alone are our supreme refuge, you are our refuge.)[10]

Around 34,500 BCE the astronomical solutions of [3] indicate that brightest Vega had already ceased to be the pole star and was located far from the north celestial pole and Earth's axial obliquity lay close to its Quaternary maximum of approximately 23.5°.It is well established that the Earth's axial obliquity oscillates between approximately 22.6° and 24.2° over a cycle of nearly 41,000 years. Around 2000 CE, the obliquity was approximately 23.44° and continues to decrease at a rate of about 46.8 arcseconds per century. Astronomical reconstruction indicates that around 34,500 BCE the Earth's obliquity was approximately 23.5°, close to the mean value of the cycle, *but on the ascending phase toward its maximum*, which would have created the impression that the Earth was sinking. The paper offers this as a plausible physical correlate of the king's "sinking" image, while noting explicitly that this connection is interpretive rather than demonstrated: the same word could equally, and more conventionally, be read as a metaphor for spiritual or cosmological decline without any specific astronomical referent, as it is by most Upaniṣadic commentators.

If the later *Maghādi* configuration is instead placed in the more recent precessional cycle around 8130 BCE, that date falls within a period of generally rising, rather than falling, global sea level [6], which the paper reads as inconsistent with the ocean-level decline implied by the MAU passage. Adding one full

[10]Maitrāyaṇīya Āraṇyaka Upaniṣad 1.4 translated by Dr. Umesh Kumar Singh.

precessional cycle shifts the *Maghādi* configuration backward to the vicinity of 34,500 BCE, where the astronomical and glaciological indicators used here are brought into better alignment. The Bṛhadratha evidence is accordingly read as supporting the older Maghā epoch over the more recent one, subject to the caveats above.

11. Regulus, Maghā, and the Boundary of Arjunī

Regulus (α Leonis), the brightest star in Leo and one of the brightest stars in the night sky, is identified in this tradition as the *yogatārā* (junction star) of Maghā. In the twenty-eight-*nakṣatra* system, Regulus is treated here as a discrete observational marker for the boundary between Maghā and Arjunī.

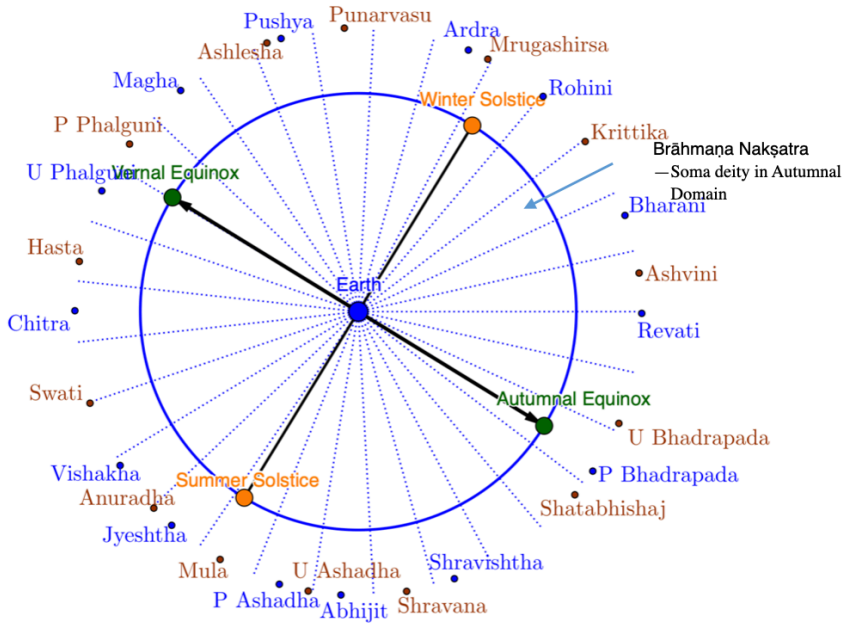

Figure 1. The 29-*nakṣatra* schema. Figure courtesy Shri Amaresh G S, GeoGebra applet.

The rate of general precession is, in the current epoch, approximately 50.3 arcseconds per year, corresponding to a full precessional cycle of approximately 25,772 years [4]. Dividing this constant by twenty-eight *nakṣatra* divisions gives approximately 920 years per division (rather than the round figure of 919 years used in the earlier, unpublished version of this argument), a refinement that does not materially change the reconstruction below.

Correcting the later *Maghādi* position (conventionally placed around 8130 BCE in some traditional reckonings) by adding one precessional cycle of 25,772 years gives:

$$8130 + 25{,}772 = 33{,}902 \text{ BCE (approximately)},$$

with the broader *nakṣatra* range extending to approximately 33,900–34,820 BCE depending on the precise precessional constant and starting epoch adopted. Within this interval, 34,500 BCE is proposed as the more specific epoch on the grounds that this is where Regulus is calculated, via Stellarium-based simulation, to mark the boundary of Maghā and Arjunī/Phalgunī[9].

Likewise, the *Arjunyādi* position (in 29 *nakṣatra* system) would be:

$$34{,}820 \text{ (Stellarium epoch of } \textit{Maghādi}) + 888 = 35{,}708 \approx 35{,}700 \text{ BCE (approx.)}$$

The reader should note that this places the vernal equinox at the relevant boundary only under the assumption that the traditional *Maghādi* starting point of 8130 BCE is itself independently reliable; that assumed starting date is carried over from the earlier unpublished study [2] and is not re-derived here from first principles, and its own basis is a further avenue for independent verification.

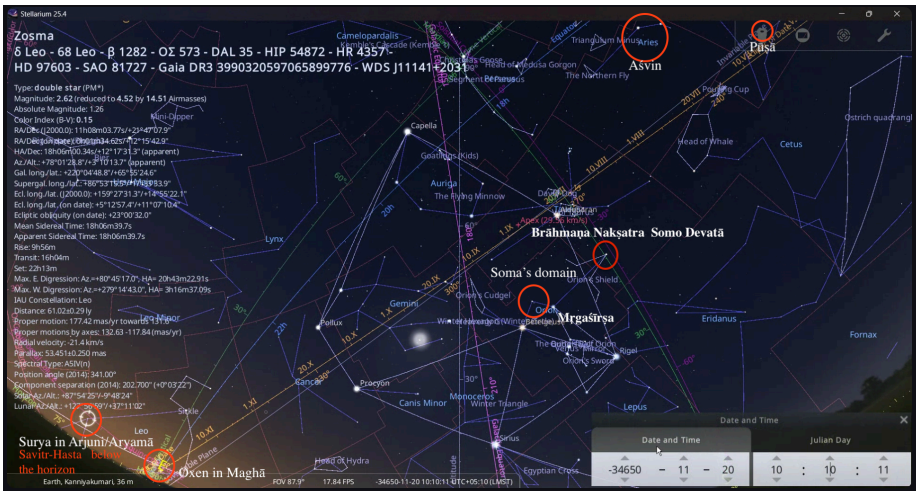

FIGURE 2. Stellarium reconstruction (34,650 BCE) showing the relative positions of Arjunī (Zosma), Maghā (Regulus), the Brāhmaṇa *nakṣatra*/Soma's domain near π^3 Orionis (Tabit), Aśvinī (β and γ Arietis), and Pūṣan (ζ Piscium). Savitā Hasta nakṣatra is below the east horizon. Figure courtesy Shri Sameer Barve; simulation generated with Stellarium open-source planetarium software [9].

12. The Aśvins and Pūṣan in the Opposite Sky

The Sūryā hymn does not end with RV 10.85.13. The next verse, RV 10.85.14, introduces the Aśvins and Pūṣan:

यदश्विना पृच्छमानावयातं त्रिचक्रेण वहतुं सूर्यायाः।
विश्वे देवा अनु तद्वामजानन्पुत्रः पितरावृणीत पूषा॥

> "When, O Aśvins, you came in your three-wheeled car soliciting the marriage of Sūryā, then all the gods assented, and Pūṣan, [your] son, chose you as his parents."

This verse is read here as consistent with a picture in which, while Sūryā moves toward Soma's domain, Pūṣan (*Nakṣatra* Revatī) is imagined at the opposite visible end of the sky, near the Aśvins, who in this reconstruction are associated with the region of Aries/Aśvinī. Figure 2 illustrates a Stellarium-based reconstruction of the relevant sky for 34,650 BCE.

13. Synthesis of the Two Arguments

The combined thesis may be summarized in six connected propositions, each offered with the qualifications noted above. First, RV 10.85.13 preserves the back-to-back *nakṣatra* sequence Aghā/Maghā and Arjunī/Phalgunī in the context of Sūryā's marriage. Second, Agni's role as leader of the procession is read as marking the vernal equinoctial point. Third, the Āśvalāyana marriage rule is read as a possible ritual continuation of the same memory. Fourth, Aryaman's association with maidens and conjugal harmony is read as consistent with the marital significance of the Phalgunī region. Fifth, the MAU Bṛhadratha material is read as a royal and cosmological witness to a *Maghādi* year. Sixth, Regulus, as the proposed *yogatārā* of Maghā, allows a precessional reconstruction to be provisionally fixed around 34,500 BCE.

In this synthesis, the Sūryā-vivāha material and the Bṛhadratha material are read as mutually reinforcing: the first supplies Ṛgvedic *nakṣatra* and marriage evidence, the second supplies *Maghādi*, pole-star, stellar-longitude, and ocean-level evidence. Their convergence is offered as support – not proof – for the hypothesis that the Maghā–Arjunī boundary was remembered, in some transmitted form, as the vernal equinox's position during a much earlier epoch than the texts' own composition.

14. Conclusion

The evidence assembled from Sūryā's bridal hymn and the Bṛhadratha tradition is presented as pointing toward a possible preserved memory of the vernal equinox in the Maghā–Arjunī region. RV 10.85.13 identifies the *nakṣatra* pair Aghā/Maghā and Arjunī/Phalgunī within Sūryā's celestial marriage; the Āśvalāyana Gṛhya Sūtra's marriage rule is read as a ritual echo of this memory; Aryaman's association with Phalgunī and conjugal order is read as compatible with the marital interpretation of the *nakṣatra* position.

The Maitrāyaṇīya tradition, through Bṛhadratha and the *Maghādi* sequence, is read as supplying a broader cosmological framework. When these data are combined with *nakṣatra* divisions, precessional calculation, and the Regulus marker, the authors propose 35,700 BCE as a candidate epoch that preserves a precise astronomical memory of the vernal equinox in the Savitā–Arjunī–Maghā region. The additional evidence of pole-star movement, stellar-longitude change, and glacio-eustatic sea-level decline is offered as consistent with, rather than as proof of, the identification of Bṛhadratha as an observer situated in this proposed epoch.

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A MATHEMATICAL MODEL FOR A SOLUTION TO HUMAN SUFFERING

PAUL CORAZZA

Abstract. The eternal Vedic wisdom teaches that all human suffering ultimately arises from loss of connection to the Source, the Self, pure consciousness. Each individual life emerges in the unfoldment of manifest existence from the hidden dynamics of the blueprint for creation: the Veda. As an individual progresses along a trajectory of living, spanning perhaps thousands of years, the natural connection to the Self may be covered over by many layers of stress, limitation, and ignorance. A simple mathematical model, based on Foundations of Mathematics, shows how individuals can be located in the universe; how individuals may indeed appear to be located very far from the Source; and how, no matter how far removed an individual may be, a return to the Source can be achieved very quickly, in just a few steps. Then, we propose a deeper mathematical model, based on recent breakthroughs in Foundations, that indicates how, from a consciousness fully awake to the unfoldment of Veda, any individual can be precisely located no matter where in time or space he or she may be. And how such a consciousness can identify the path of return for any individual, based on unmanifest dynamics of the Veda that are responsible for return to the Source. This deeper mathematical model arises from the recent discovery that the universe of mathematics is equipped with a companion map that embodies the transformational dynamics underlying the unfoldment of every detail of the mathematical universe and how each object is related to its Source.

1. Introduction

The vision of the Vedic sages tells us that human suffering arises when individual awareness is not established in its universal value, pure consciousness. Maharishi Mahesh Yogi—a modern-day exponent of Vedic wisdom and creator of the Transcendental Meditation® program—writes, in his commentary to the *Bhagavad-Gita* ([10, p. 78]):

> All problems of life arise from... the mind's ignorance of its own essential nature, which is universal and the source of infinite energy and intelligence. This ignorance of one's own self is the basis of all problems, sufferings, and shortcomings in life.

According to Maharishi ([12]), the field of pure consciousness is universal in its character; it underlies the activity not only of thoughts, feelings, and sensations but all activities of the universe. The laws governing human thought, and indeed all the laws of nature, have their root and starting point in the field of pure consciousness. Thus, when individual awareness becomes identified with pure consciousness, the most fundamental level of nature's functioning becomes enlivened in the individual's life; the desires and thoughts that arise in such a mind naturally express nature's

intention. And, as Maharishi points out, the all-powerful intelligence underlying nature's functioning, expressed as laws of nature, that is capable of effortlessly maintaining the unimaginable complexity of the entire universe without a problem, is more than capable of bringing to fruition the desires and intentions of any individual. The only prerequisite for living such a life supported by natural law is that individual life be lived in accord with the laws of nature. The laws of nature are the pathways along which nature's mighty power flows; to take advantage of this flow of nature, one's life must move in step with nature, and this is accomplished simply by allowing individual awareness to become established at the level at which these laws begin to operate, the field of pure consciousness.

A reasonable concern is whether this goal of reconnecting to one's source is a *realistic* goal. As Lord Krishna explains[1] to Arjuna in the *Gita* (IV.5):

> Many births have passed for Me and for you also, O Arjuna. I know them all but you know them not...

His words tell us that, for many, one's path entails many lifetimes. And for most, ignorance of the Self has been the reality through this entire expanse of time. How then can this long-term separation be transformed so easily?

Fortunately, as the *Gita* declares[2] (II.40), "no obstacle exists" in the path to reuniting with the Self. Maharishi, in his commentary to verse 45 of Chapter II of the *Gita*,[3] describes the process in this way, applicable to anyone anywhere ([10]):

> When the Lord says: "Be without the three gunas," He... means that in whichever field of the three gunas you have your stand, from there you are to begin moving towards subtler planes of the gunas and, arriving at the subtlest, come out of it, transcend it, be by yourself, "possessed of the Self"...

Simply put, no matter who or where one is, no matter how long the journey has been, one starts the path to the Self wherever one is; in every case, the steps are few and effort is minimal.

Maharishi points out ([10, pp. 118–120]) that the reason this path is inherently effortless is that the inward dive[4] that leads to pure consciousness is governed by the natural tendency of the mind to seek fulfillment; the mind naturally moves toward a field of greater bliss and fulfillment without effort, if given a chance to do so. Once the practice begins, only a few steps are needed for the transcendental value of consciousness, beyond the finest levels of thought and perception, to be experienced.

Maharishi explains ([12, pp. 97–98]), "As we gain more and more familiarity with that self-referral performance [of pure consciousness], our thoughts and actions spontaneously begin to be as orderly and evolutionary as all the activity of nature."

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[1] See [10, p. 259].

[2] See [10].

[3] An excerpt of Maharishi's translation of this verse follows:

> Be without the three gunas, O Arjuna, freed from duality... possessed of the Self.

[4] In the practice of Transcendental Meditation.

These steps of progress result in winning the support of total natural law for one's life, ambitions, and well-being.

It is possible to provide a mathematical model of these steps of transformation. Doing so can provide some clarification and a technical validation of the process.

2. A Simple Mathematical Model

Our mathematical models are based on the characteristics of the *universe of mathematics*. We therefore begin with a brief introduction to this universe and some basic related notions.

The universe of mathematics is understood to contain all the "mathematical objects" that arise in mathematics. Mathematical objects include[5] number systems (like the set $\mathbb{W} = \{0, 1, 2, \ldots\}$ of whole numbers), operations (like addition and multiplication of numbers), sets, functions (mappings), geometric objects (points, lines, spheres), and more advanced structures used in mathematics (groups, rings, topological spaces). Some important terminology related to mathematical objects is the following: Whenever a mathematical object x belongs to a collection A of objects, x is called an *element of* A and one writes $x \in A$. Also, the set that has no elements is called the *empty set* and is denoted $\emptyset$.

It was recognized 150 years ago that every mathematical object can be represented as a set.[6] An important example of this fact, which we will need in our discussion here, is the way whole numbers are represented as sets:

$$0 = \emptyset, \quad 1 = \{0\} = \{\emptyset\}, \quad 2 = \{0, 1\}, \quad \ldots \quad n = \{0, 1, \ldots, n-1\}, \ldots$$

This discovery leads to the consequence that the universe of mathematics can be understood as a universe of *sets*—usually denoted V. However, for this viewpoint to be meaningful, the notion of *set* needs to be sufficiently rigorous. In the early days of mathematical foundations, however, only a loose definition of *set* (namely, a set is any collection of objects) was used.[7] This loose definition led to logical inconsistencies, and so, in order to precisely indicate the meaning of *set*, axioms were developed that specified which collections should be viewed as sets—as elements of the universe V. These axioms are called the *Zermelo-Fraenkel axioms with the Axiom of Choice,* or ZFC.[8] The development of these axioms provided further confirmation of the universality of set theory: It was seen that every theorem in

[5] A catalog of mathematical objects, including examples and definitions, is given in the Appendix, Section A.1.

[6] See the Appendix, Section A.2, for a discussion on this point, together with examples.

[7] A full discussion of this early conception of sets can be found in [7].

[8] A complete list of these axioms, together with a discussion of the inconsistency that arises from the early definition of set, can be found in [8] and is conveniently available online at `https://en.wikipedia.org/wiki/Zermelo-Fraenkel_set_theory`.

mathematics can be restated in terms of the language of sets and derived directly from the axioms of ZFC.

Here are three examples of ZFC axioms:

- *Axiom of Empty Set.* There is a set with no element.
- *Axiom of Pairing.* For any sets X, Y there is a set whose only elements are X, Y (denoted $\{X, Y\}$)
- *Power Set Axiom.* For any set X there is a set, denoted $\mathcal{P}(X)$, whose elements are precisely the subsets of X.

These axioms—like all of the ZFC axioms—provide important information about the contents of V. The Axiom of Empty Set tells us that the empty set is an element of V. The Axiom of Pairing tells us that, if X and Y are elements of V, then so is $\{X, Y\}$. And the Power Set Axiom tells us that, if X is in V, so is its power set $\mathcal{P}(X)$.

The axioms collectively tell us how to build V: V is built in stages $V_0, V_1, V_2, \ldots$. The zeroth stage V_0 is defined to be the empty set $\emptyset$. Each subsequent stage is obtained from the previous stage by an application of the power set operator $\mathcal{P}$. Since the only subset of $\emptyset$ is $\emptyset$ itself,

$$V_1 = \mathcal{P}(V_0) = \mathcal{P}(\emptyset) = \{\emptyset\}.$$

Continuing in this way, the first stages of V are defined as follows (Figure 1):

$$\begin{aligned} V_0 &= \emptyset \\ V_1 &= \mathcal{P}(V_0) = \{\emptyset\} \\ V_2 &= \mathcal{P}(V_1) = \{\emptyset, \{\emptyset\}\} \\ V_3 &= \mathcal{P}(V_2) = \big\{\emptyset, \{\emptyset\}, \{\{\emptyset\}\}, \{\emptyset, \{\emptyset\}\}\big\} \\ \cdot &= \cdot \\ \cdot &= \cdot \\ \cdot &= \cdot \end{aligned}$$

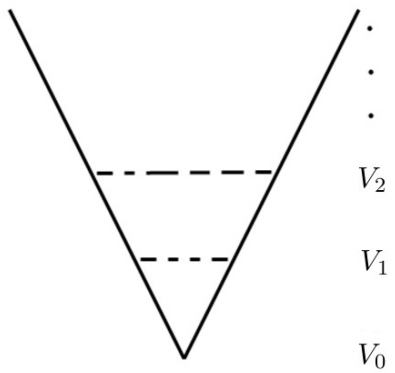

FIGURE 1. First Stages of the Universe V

The indices $0, 1, 2, \ldots$ of these stages $V_0, V_1, V_2, \ldots$ are the familiar whole numbers; in the present context they are called *finite ordinals*. To complete the construction of V, it is necessary to introduce further stages. The indices for these additional stages are called *infinite ordinals*.[9] For any infinite ordinal α, the αth stage is defined[10] by

$$V_\alpha = \begin{cases} \mathcal{P}(V_\beta) & \text{if } \alpha \text{ is the immediate successor of } \beta \ (\alpha = \beta + 1) \\ \bigcup_{\beta<\alpha} V_\beta & \text{if } \alpha \text{ has no immediate predecessor.} \end{cases} \tag{2.1}$$

The full universe V is obtained by forming the union of all these stages (Figure 2).

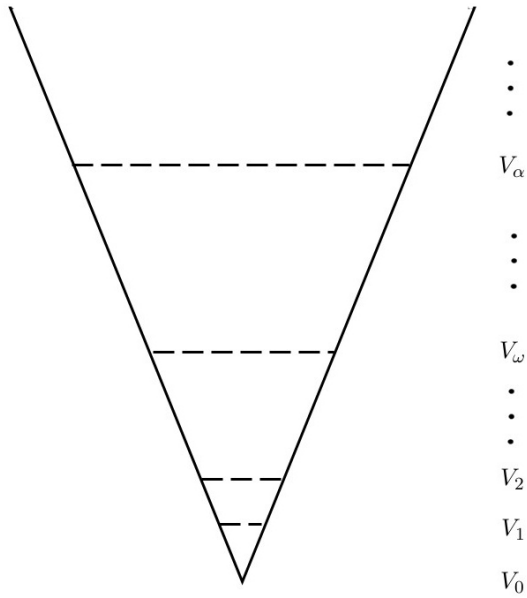

FIGURE 2. Full Universe of Sets

An instructive example of these stages arises at the stage ω: The ordinal ω denotes the smallest infinite ordinal that is bigger than all the whole numbers. Therefore, since ω has no immediate predecessor (if $n < \omega$, then $n+1$ is also less than ω and $n < n + 1$), we have, by definition (using the second line of equation (2.1)):

$$V_\omega = \bigcup_{n<\omega} V_n = V_0 \cup V_1 \cup \cdots \cup V_n \cup \cdots .$$

The ordinal that comes immediately after ω is $\omega+1$, and so we have (using the first line of equation (2.1)):

$$V_{\omega+1} = \mathcal{P}(V_\omega).$$

Infinite ordinals play an important role in the development of mathematics. One important example is the fact that many of the infinite ordinals are also *cardinal*

[9]Ordinals are discussed in more detail in the Appendix, in Section A.3.

[10]Section A.4 in the Appendix goes into more detail about how the stages are defined.

numbers. Cardinal numbers indicate the size of a set. For instance, the cardinal number 3 is used to tell us the size of the three-element set $\{2, 8, 25\}$; one writes $|\{2, 8, 25\}| = 3$. Among the infinite ordinals, the first one that is also a cardinal number is ω; ω happens to be the size of the set $\mathbb{W}$ of whole numbers—one writes $|\mathbb{W}| = \omega$. The cardinal ω is also written ω_0 suggesting the beginning of a long list. One can list all the infinite cardinals in the following way:

$$\omega_0, \omega_1, \omega_2, \ldots.$$

Each successive cardinal in this list is a bigger size[11] of "infinity" than the previous one. For instance, if $|X| = \omega_1$ and $|Y| = \omega_2$, then the set Y has bigger infinite size than the set X. Similarly, once the unfoldment of V reaches V_ω, the sizes of the stages increase dramatically, with each successive stage representing a bigger infinite size than the previous one. For instance, $|V_\omega| < |V_{\omega+1}|$.

As mentioned above, both of our models are based on characteristics of V. The universe V provides a natural mathematical analogy for the totality of existence: V is the container of all the individuals that make up the universe, yet is not itself one of those individuals.

Our first model probes the structure of the stages of V. As we have seen, all sets arise in the unfoldment of the stages $V_0, V_1, \ldots$; since the process begins with the empty set (namely, V_0), we take the empty set to be the *source* of all sets in the universe.

Next we examine more closely how sets arise in the universe in the first place. Since each successive stage consists of all the subsets of the previous stage, in order for a set to appear as an element of a particular stage, it must first occur as a subset of the previous stage. For instance, the set $\{1\}$ occurs for the first time in the stage V_3 since it first arises as a *subset* of the previous stage V_2.[12] Speaking intuitively, one could say that a mathematical object that is *preparing to be born* as a set in V starts out as a *subset* of a particular stage V_α, and then achieves formal set-hood in the subsequent stage $V_{\alpha+1}$.

We can locate sets in the universe by specifying their *rank*. The rank of a set x is the smallest ordinal α for which x is a subset of V_α; one writes

$$\text{rank}(x) = \alpha.$$

It is easy to check that $\text{rank}(0) = 0$ and $\text{rank}(1) = 1$. In general, for any ordinal α, $\text{rank}(\alpha) = \alpha$.

There is a mathematical way of determining where a given x is to be found in V using a formula for computing ranks:[13]

$$\text{rank}(x) = \sup\{\text{rank}(y) + 1 \mid y \in x\}. \tag{2.2}$$

When we compute the rank of a set—its location in V—we discover that, typically, x is very far removed from the starting point V_0. This is because of the rapid growth rate of the sizes of the stages. One might expect therefore that any path of

[11]The fact that there are many sizes of infinite sets was established by Cantor; see [8].

[12]Details for this example are worked out in the Appendix in Section A.4.

[13]Section A.4 in the Appendix provides a discussion about ranks, including examples and more details about the formula (2.2).

return from x to $\emptyset$ would be extremely long. Nevertheless, as we now show, there is a quick and simple procedure by which x can indeed return to $\emptyset$.

To understand this procedure, we need to observe one special characteristic of sets: For any nonempty set x, consider the following procedure:

> *Forming a Descending Chain of Sets.* Pick an element y_0 of x; if y_0 is also nonempty, pick an element y_1 of y_0. Continue, to obtain
>
> $$\cdots \in y_2 \in y_1 \in y_0 \in x.$$

The important fact is that this process cannot continue forever; there is no *infinite* descending chain starting at x.[14] One says that *sets in V are always well-founded.*

> *Fact.* For every x in V, there is no infinite descending chain starting at x; that is, x is well-founded.

Now this Fact is just what is needed to obtain a short path for any set x to "return to its source": Given a nonempty set x, form a descending chain as in the procedure above:

$$\cdots \in y_2 \in y_1 \in y_0 \in x.$$

However, as the Fact tells us, this path cannot be infinitely long, so it must terminate after finitely many steps; let us call the final set in this chain y_n. Since the process cannot be continued further, y_n must be empty. We have:

$$\emptyset = y_n \in y_{n-1} \in \cdots \in y_2 \in y_1 \in y_0 \in x. \tag{2.3}$$

We have obtained a finite path for x to return to its source $\emptyset$ (Figure 3).

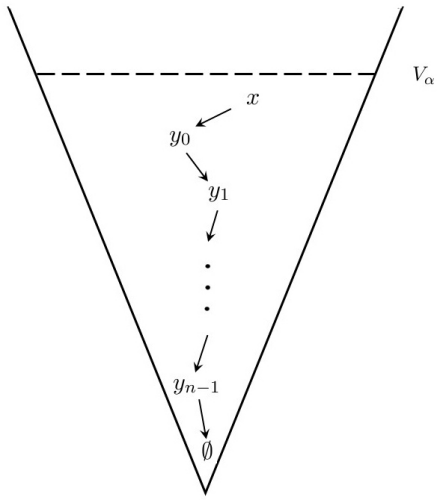

FIGURE 3. The Path of Return for an Individual x

We have seen therefore how, in this first mathematical model, each individual x in the universe can be located in the universe—via its rank—and, no matter how

[14]Appendix A.4 provides a proof of this fact.

far removed x may be from the source (the empty set $\emptyset$), there is a path for x to return to $\emptyset$ that involves only finitely many steps, as in the chain (2.3).

3. A Rich Mathematical Model

Our first mathematical model illustrated the principle that, no matter how far removed an individual may appear to be from his or her source, returning to the Self can be achieved in just a few steps.

Our second mathematical model considers this principle from a more universal perspective. At some point in the evolution of an individual, a desire arises to reconnect to his or her Self. The question of how to do this may arise. Or, even if a good method is found, another question that may arise is "How long does it take to fully reconnect to my higher nature?" and "Am I making progress?" These are questions that arise from the individual perspective. But it is possible to view all questions about individual life and its path to the Source from a deeper perspective.

The Vedic wisdom declares that the Veda is the blueprint of creation. The unmanifest dynamics of pure consciousness take shape as the fundamental frequencies that structure the Veda and, ultimately, manifest existence. These frequencies arise, Maharishi observes, in the collapse of the total value of wholeness to a point, as expressed in the following *richo akshare* verse from Rig Veda:[15]

> *Richo akshare parame vyoman*
> *yasmin deva adhi vishve nisheduh*
>
> The verses of the Veda exist in the collapse of fullness (the *kshara* of 'A') in the transcendental field, self-referral consciousness, the Self, in which reside all the Devas, the impulses of Creative Intelligence, the Laws of Nature, responsible for the whole manifest universe.

In Maharishi's *Apaurusheya Bhasya* (uncreated commentary), he explains ([14, p. 93 and pp. 495–500]) how these fundamental frequencies unfold in successive packets of total knowledge, from the first letter "A" to the first syllable "AK" to the first *richa*, to the first *sukta*, and to the first *mandala* of Rig Veda, and then finally in its most elaborated form as the eightfold *prakriti*, which then structures the manifest universe.

As indicated in the *richo akshare* verse displayed above, the first step in this sequential unfoldment is the collapse of the unbounded totality represented by "A" to the point value "K" located within wholeness. In this collapse, the point "K" is infused with infinite dynamism sufficient to drive the full expression of manifest existence [13, p. 32]:

> Vedic Science identifies K as the point of all possibilities...from which the whole sequential progression of the samhita emerges simultaneously. This locates in K the infinite creative potential of nature that in one stroke can give expression to the infinite diversity of creation.

[15] Rig Veda I.164.39. See [1, pp. 180–181].

As this quote indicates, the unfoldment of Veda and manifest existence from the point "K" is both sequential and *simultaneous*; from the deepest perspective, it all happens at once.

An awareness that can access the mechanics of this unfoldment can, Maharishi explains, know every detail of an individual's path to full awakening ([11, pp. 20–21]):

> There is nothing in the cosmos, either in the material plane or in the spiritual plane which cannot be directly cognized. Vedas provide a direct method of direct cognition of material existence in this unlimited cosmos, and they provide a direct technique to cognize that which is evenly permeating, evenly pervading the entire physical structure of the cosmos.

The level of intelligence that can know all details of all levels of existence is called by Maharishi *Jyotish-mati pragya* ([16]):

> This intelligence is called Jyotish-mati pragya—the intelligence that is completely awake within itself and aware of all the infinite possible structures contained within its own value.... When this level of awareness is completely open to itself, it knows all about everything that is contained within its own structure. It is from here that all knowledge about everything in creation is available.

Our second mathematical model gives expression to this higher type of knowing regarding the emergence and path of individual life. In our first model, the location of an individual x in V can be computed using a recursive formula, and the return of x to its source V_0 is seen to involve only finitely many steps. Our second model presents the more universal view that the location of each set x and the full path of return to the source can be known directly, in an instant. This is achieved by making use of subtler mechanics inherent in V, analogous to the deeper field of intelligence, *Jyotish-mati pragya*. This approach reveals the deeper insight that each set arises from and returns to its source by way of the self-interacting dynamics of V, involving no steps at all.

These alternative accounts of the unfoldment of sets are analogous to the different accounts of the origin of the physical universe. The modern physics account of the emergence of the universe is found in the Big Bang theory,[16] according to which the universe began around 13.8 billion years ago as an expansion from an initial hot, dense, and infinitely small point. A subtler account (described above) is found in the Vedic perspective according to which the manifest universe arises from the dynamics of pure consciousness as it unfolds the structuring impulses of the universe in the form of the sequential and simultaneous unfoldment of the Veda from the point "K." Both accounts may be said to be valid at their own level.

To present our second model, we introduce a modern view concerning the dynamics of the set-theoretic universe V. Historically, the universe of sets has been seen as simply the vast collection of all sets needed to develop mathematics. A more recent view ([2] and [4]), however, is that V, as a model of wholeness, has "unmanifest dynamics." The recognition that such dynamics should be present

[16]See for example the Wikipedia article `https://en.wikipedia.org/wiki/Big_Bang`.

within V emerged as an insight on the path to solving a long-standing problem in mathematical foundations concerning the existence of certain very large and exotic infinities called *large cardinals*. On the basis of the standard ZFC axioms, it is impossible to account for large cardinals,[17] though they nevertheless mysteriously appear in the solution of a wide range of research problems.[18] However, once the inherent dynamism within V, suggested by recent research, is given mathematical expression in the form of a truth-preserving map $j : V \to V$, called a *nontrivial elementary embedding*,[19] then the very strong properties of large cardinals can be accounted for.

To say that a map $j : V \to V$ is an elementary embedding means that all properties and relationships[20] that hold true in V continue to hold true after j is applied. For instance, if x, Y, and Z are sets belonging to V, and x is a member of Y, and the size of Y is less than the size of Z, then these relationships hold after j is applied; that is, $j(x)$ is a member of $j(Y)$, and the size of $j(Y)$ is less than the size of $j(Z)$.

This truth-preserving characteristic of such a map j makes it a natural analogue to the unmanifest dynamics of pure consciousness, of wholeness: First, the behavior of j is *unmanifest* in the sense that j itself is not a member of V. Secondly, because all relationships are preserved by j, one can say that V is unchanged by its transformational dynamics. This is a vital feature of the dynamics of pure consciousness: Though the dynamics of pure consciousness give rise to all manifest existence, pure consciousness itself remains unchanged by these dynamics. Maharishi ([15, p. 12]) describes this phenomenon in the following way:

> Pure consciousness, pure Being, is maintained as pure consciousness and pure Being all the time, and yet it is transformed into all the different forms and phenomena. Here is the cosmic law, one law which never changes and which never allows absolute Being to change. Absolute Being remains absolute Being throughout, although it is found in changed qualities here and there in all the different strata.

Once the embedding j is introduced and properly formalized as a new axiom of set theory, new dynamics within V become apparent. The requirement that j be *nontrivial* means that some set in V is moved by j: For some z, $j(z) \neq z$. And it can be shown that in fact some *ordinal* must be moved by j. The first ordinal moved by j is called the *critical point of* j, and it is denoted κ ("kappa"); κ exhibits [3] all the very strong properties of the best-known large cardinals. In this way, simply by giving formal expression to the presence of underlying dynamics of V, most of the long-standing mystery concerning existence of large cardinals is solved. Figure 4 helps to visualize the behavior of j: All sets that lie in the stages before V_κ are not moved by j; j is "silent" before its first "stirring" when it is applied to κ.

The existence of such an embedding j has other important consequences. From the dynamics of j, as it interacts with its critical point κ, emerge a variety of other

[17] See for example [8] and further discussion in the Appendix, Section A.6.

[18] Examples of this phenomenon are given in [4].

[19] See Section A.6 in the Appendix.

[20] Technically speaking, all *first-order* properties and relationships are preserved.

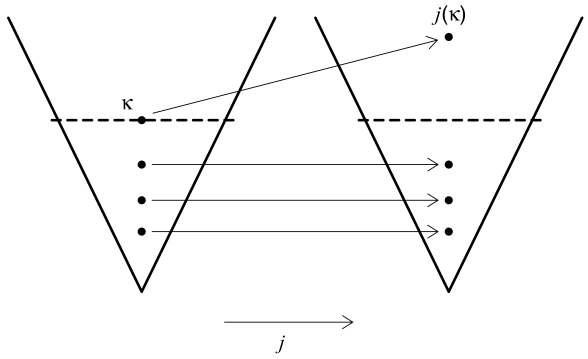

FIGURE 4. Silent Below κ, Dynamic at κ

elementary embeddings that give partial expression to j—maps of the form $i : V_\eta \to V_\xi$, known as *extendible embeddings*. These, through strongly self-referral dynamics, produce[21] a special function $f : \kappa \to V_\kappa$, whose behavior can be represented as a sequence

$$f = \langle z_0, z_1, \ldots, z_\alpha, \ldots \rangle \tag{3.1}$$

that is contained as a subset of the κth stage V_κ of V. This sequence, called an *extendible Laver sequence*, is a natural analogue to the Veda in the context of the universe of sets: From f all sets in the universe can be seen to emerge (Figure 5).

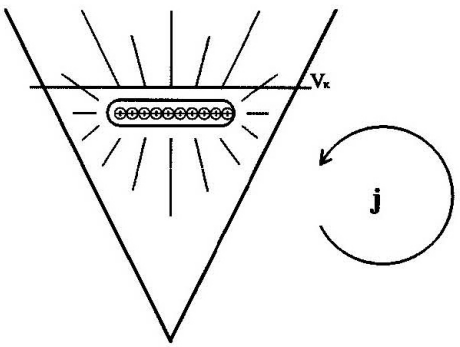

FIGURE 5. Embedding $j \Rightarrow$ Laver Sequence $\Rightarrow$ All Sets

To see how this is possible, we mention one of Maharishi's insights into the structure of the Veda and Vedic literature. He observes [17, pp. 25–26] that there is a part of the Veda that is concerned with expansion from point to infinity and another that is concerned with the return of parts to wholeness.[22] It is therefore

[21] See Section A.7 in the Appendix.

[22] The aspect of the Veda concerned with expansion, according to Maharishi, is *Vyakarana*, and the aspect concerned with return is *Nirukta*.

natural to see this Laver sequence f, being a blueprint for the unfoldment of all sets in V, as an expression of this *expansive* aspect of the Veda. As we will explain in a moment, there is a natural variant of f, denoted f^{op}, that gives expression to the *returning* aspect of the Veda.

To locate a particular set x in V, making use of the Laver sequence f, one applies an extendible embedding i to f with reference to the starting point[23] κ. One obtains

$$x = i(f)(\kappa). \tag{3.2}$$

Intuitively speaking, x is "instantly" located by applying this "high-level" intelligence i—which arises directly from the unmanifest dynamics of pure consciousness represented by j—to the mathematical representative f of the Veda.

Dually, we find a similar process in locating the path of x to "full integration" with its source κ. To see the "pathless path" in the return of x to κ, we call upon this same embedding i again and now apply it to the dual (*returning* aspect) f^{op}, with respect to the object x. One obtains the following dual formula:[24]

$$\kappa = i(f^{\mathrm{op}})(x).^{25} \tag{3.3}$$

Our second model, therefore, gives expression to the life journey of individuals from the universal perspective—a perspective that is accessible from an awareness established in *Jyotish-mati pragya*. This perspective tells us that one's entire journey from the beginning until the present moment can be appreciated in its fullness in a single moment of cognition. And, likewise, the path of return leading to full awakening can also be appreciated in a single moment of knowing. Our model represents these dynamics with simple "no step" formulas: Locating an individual x, wherever and whenever he or she may exist, is accomplished by

$$x = i(f)(\kappa).$$

And a snapshot view of the individual's path to full awakening is modeled by

$$\kappa = i(f^{\mathrm{op}})(x).$$

[23]In the present context, because of equation (3.2), κ (rather than $\emptyset$) is understood to be the starting point or source of all sets.

[24]This technique works only for those x that lie outside of the κth stage V_κ (the reason for this is explained in the Appendix; see section A.7). This is not much of a restriction since V_κ can be viewed as another way of expressing the starting point κ; indeed, V_κ is definable from κ.

[25]This immediate knowledge of the "pathless path" of any individual in his or her journey to full awakening becomes possible at the highly refined level of awareness called *Jyotish-mati pragya*. What are the precise mechanics of this type of knowing? Our model offers an analysis that illustrates the phenomenon, summarized in the equation $i(f^{\mathrm{op}})(x) = \kappa$, but we cannot expect that this conceptual analysis captures the full reality of it. Indeed, as Maharishi points out, mathematical models necessarily fall short of the reality itself ([14, p. 558]):

> [The dynamics of pure consciousness are]...unknown to the mathematics of the conceptual world, unknown to the mathematics of the world of diversity, which has its basis in the notion of reality—not the reality but the notion about it, the concept of it. Born of a notion means born not of reality but of the idea of reality, the shadow of reality... .

4. Conclusion

We began our discussion with the age-old question: What is the cause of human suffering? The answer, provided by the eternal Vedic wisdom, is that suffering arises from lack of connection to one's source, to the Self. It is reasonable, however, to suppose that the way back to the Self would be a long, arduous process, considering the fact that this deepest aspect of one's life is typically buried beneath lifetimes of challenges, stress, and mistakes. But Lord Krishna proclaims that, in this return journey to the Self, "no obstacle exists," and Maharishi's commentary on the *Gita* makes it clear that since "who one is" can never really be lost, the path of return involves only a few steps and minimal effort.

We have modeled the life journey of the individual from beginning to end in two different ways. In both models, we treat the universe V of sets as an analogy both for the physical universe and for *wholeness*. In both models we consider how an individual x is born in V in relation to the starting point of emergence of all objects, and the mechanics by which x can be said to return to this starting point.

Our first model describes the emergence of individual objects x in the universe: Each begins as a subset of one of the stages V_α in the build-up of V and then "takes birth" as an actual object in the next stage $V_{\alpha+1}$. Because there are vastly more sets in V in the later stages of its unfoldment, an individual will typically be located very far from its source, which in this model is V_0—the empty set. The location of any x in the universe, which is called the *rank* of x, can be computed by the formula: $\text{rank}(x) = \sup\{\text{rank}(y) + 1 \mid y \in x\}$.

This model shows how x's path of return to $\emptyset$ necessarily involves very few steps, despite x's apparent remoteness from the starting point: For each x there is a chain of sets starting at x that leads back to $\emptyset$ in just *finitely many steps*:

$$\emptyset = y_n \in y_{n-1} \in \cdots \in y_2 \in y_1 \in y_0 \in x.$$

These elements of the model give expression to the insight that the path of return to one's Self is inherently not too long, and obstacle-free.

Our second model provides insight into a much broader perspective concerning the journey of individuals toward full awakening. Our first model focused on the human condition of (apparently) being remotely situated from one's source and the encouraging insight from the *Gita* that, no matter who or where one is, experience of the transcendental value of awareness is just a few steps away. Our second model emphasizes the universal perspective according to which the emergence of individual consciousness and its full awakening to the Self occur as part of the sequential and *simultaneous* unfoldment of the Veda itself, from its first syllable "AK" through its first *richa*, first *sukta*, and first *mandala*, ultimately to the manifest universe. These simultaneous transformational dynamics can be appreciated by a highly refined quality of intelligence, *Jyothish-mati pragya*: Each individual journey can be cognized, from beginning to full awakening, in an instant.

In this second model, the unmanifest dynamics from which everything occurs in an instant is modeled by the unmanifest transformational dynamics of V itself, captured in the notion of an *elementary embedding* j from V to itself. In these dynamics, a point κ within V is first located—this κ is the first ordinal moved by j,

representing the first stirring within V from silence.[26] As j interacts with κ, partial expressions of j called *extendible embeddings* arise; these embeddings are analogues to the impulses of intelligence within the transcendental field. From these, a special sequence $f : \kappa \to V_\kappa$—a *Laver* sequence—is encoded at stage V_κ, which serves as a blueprint for all sets in the universe. Such a Laver sequence provides an analogy for the Veda itself. Since κ is the starting point for the emergence of f and from f all sets can be seen to arise, in this model we consider the starting point, and point of return, for any x to be κ itself.

The instantaneous cognition of x's full journey is represented in this model in the formulas

$$x = i(f)(\kappa) \qquad \text{and} \qquad \kappa = i(f^{\text{op}})(x).$$

The emergence of x in the universe—indicated in the first formula—is seen to arise as a result of application of the appropriate extendible embedding to f, at the starting point κ. Here, f represents the aspect of the Veda concerned with *expansion*. When higher intelligence interacts with the Veda relative to the starting point κ, x is found in V in an instant. Likewise, in the second formula, when this same "impulse of intelligence"—represented by i—is applied to f^{op} with respect to the individual x (here, f^{op} represents the *return* aspect of the Veda), the starting point κ appears, again, in an instant.

The first model can be seen to reassure any seeker that the path to freedom is indeed available to him or her, and that this path is an accessible one. The second model gives further assurance that the path to full awakening, from a deeper perspective, has already been completed—success on this vital journey is guaranteed.

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[26]The map $j : V \to V$ is "silent" through the stages $V_0, V_1, \ldots, V_\beta, \ldots$, for $\beta < \kappa$ in the sense that, for each set y in any of these stages, $j(y) = y$; in other words, j does nothing to the sets in stages before the κth.

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Appendix A. An Introduction to the Underlying Mathematics

This Appendix provides technical details with examples and additional explanations for some of the topics in the main text.

A.1. **A Catalog of Mathematical Objects.** Mathematical objects include any kind of structure, operation, or relation that occurs in mathematics. Here are some examples relevant for this article:

(1) Number systems, like the set of natural numbers $\mathbb{N} = \{1, 2, 3, \ldots\}$, the set of whole numbers $\mathbb{W} = \{0, 1, 2, 3, \ldots\}$, the set of rational numbers (like $2/3$ and $-1/8$), and the set of real numbers (like $\sqrt{2}$ and π).
(2) Operations, like addition and multiplication, defined on number systems or on other mathematical structures.
(3) Sets, like $\{1, 4, 5\}$ and the *empty set* $\emptyset$—the set that has no elements. Loosely speaking, a set is simply a collection of mathematical objects; objects that belong to a set are called *elements* of the set. The notation $x \in A$ signifies that the mathematical object x is an element of the set A.
(4) Operations and relations on sets. Set operations include *union* ($\cup$) and the *power set* operation ($\mathcal{P}$). The two most significant relations on sets are the *membership relation* ($\in$), which indicates when an object belongs to a set (as described in the main text), and the *subset* ($\subseteq$) *relation*. We elaborate further below with definitions and examples:
   (i) The union of sets A and B (denoted $A \cup B$) is the set consisting of all elements that belong either to A, to B, or to both. For example,
   $$\{1, 2, 4\} \cup \{2, 3, 7\} = \{1, 2, 3, 4, 7\}.$$
   (ii) A set A is said to be a *subset* of a set B if every element of A is also an element of B, and in that case one writes $A \subseteq B$. For instance, $\{2, 3, 7\} \subseteq \{2, 3, 7, 11\}$.
   (iii) The *power set* of a set A is the set whose elements are all the subsets of A; this set of subsets is denoted $\mathcal{P}(A)$. Note that the empty set $\emptyset$ is a subset of every set.[27] We give a simple example. Let A denote the

[27]The reasoning that proves this is the following: Suppose A is a set and $\emptyset$ is not a subset of A. By definition of subset, there must exist an element of $\emptyset$ that is not an element of A. But no

set $\{1, 2\}$. Then the power set of A is given by:

$$\mathcal{P}(A) = \big\{\emptyset, \{1\}, \{2\}, \{1, 2\}\big\}.$$

(5) Functions (also called mappings). A function f from set A to set B, denoted $f : A \to B$, associates with each element of A a unique element of B. For example, consider the mapping that associates to each natural number n the number n^2. Since f takes elements from $\mathbb{N}$ to elements of $\mathbb{N}$, one writes

$$f : \mathbb{N} \to \mathbb{N} \text{ is defined by } f(n) = n^2.$$

(6) Geometric objects, like points, lines, triangles, circles, and spheres.
(7) Structures used in the sciences, like groups, rings, topological spaces, and Hilbert space, to name a few.

A.2. **Mathematical Objects As Sets.** We give examples showing how mathematical objects are represented as sets.

Ordered pairs. An ordered pair has the form (a, b) where a and b are mathematical objects. Ordered pairs are similar to sets that have just two elements, except that the *order* in which the elements occur in an ordered pair is taken into account (but not in an ordinary set). For instance, whereas the sets $\{1, 2\}$ and $\{2, 1\}$ are equal (since they have the same elements), the ordered pairs $(1, 2)$ and $(2, 1)$ are *not* equal since the numbers 1 and 2 occur in different order. The defining characteristic of ordered pairs, therefore, is given by the following criterion:

> *Criterion for Equality of Ordered Pairs.* Ordered pairs (a, b) and (c, d) are *equal* if and only if $a = c$ and $b = d$.

If one defines ordered pairs as sets in the following way:

$$(a, b) = \big\{\{a\}, \{a, b\}\big\},$$

it is not hard to show that, under this definition, the Criterion of Equality is satisfied. In particular, one shows the following:

> For all a, b, c, and d,
>
> $$\big\{\{a\}, \{a, b\}\big\} = \big\{\{c\}, \{c, d\}\big\} \text{ if and only if } a = c \text{ and } b = d.$$

Functions. If f is a function that assigns values in a set A to values in a set B—denoted $f : A \to B$—then f may be represented as a set of ordered pairs in the following way:

$$f = \{(a, b) \mid b = f(a)\}. \tag{A.1}$$

Then since each ordered pair is representable as a set, we have represented any such function as a set as well.

Using an example from the main text, suppose $f : \mathbb{N} \to \mathbb{N}$ is defined by $f(n) = n^2$. Using the representation given in equation (A.1), we have

$$f = \{(n, r) \mid r = f(n)\} = \{(n, n^2) \mid n \in \mathbb{N}\}.$$

---

such element could exist since $\emptyset$ has no elements. Therefore, the assumption that $\emptyset$ is *not* a subset of A is incorrect.

As shown in the example above, each ordered pair (n, n^2) from f is itself a set. Formally, therefore, one can represent this function by

$$f = \Big\{ \{\{n\}, \{n, n^2\}\} \mid n \in \mathbb{N} \Big\}.$$

Addition of Natural Numbers. Certainly addition of natural numbers—like $2 + 3 = 5$—is fundamental to mathematics, but it may not be obvious that addition, too, is a mathematical object, representable as a set. To see this, we first introduce the concept of the *Cartesian product* of two sets: The Cartesian product of sets A and B, denoted $A \times B$, is defined by

$$A \times B = \{(a, b) \mid a \in A \text{ and } b \in B\}.$$

Since $A \times B$ consists of ordered pairs, which are themselves sets, it is easy to see that $A \times B$ is also a set.

Now we give a *name* to the operation of addition of natural numbers—we call it Add. Add is to be understood as a function which behaves like this: $\text{Add}(m, n) = m + n$. Therefore, we can express addition in the following way:

$$\text{Add} : \mathbb{N} \times \mathbb{N} \to \mathbb{N} \text{ defined by } \text{Add}(m, n) = m + n.$$

We see from this formulation that addition of natural numbers is, formally speaking, a function, and functions, as we have already seen, can be represented as sets.

A.3. **The Ordinal Numbers.** Ordinal numbers are used to specify arrangements of objects in a particular order. A simple example is an arrangement of five desks in a classroom:

$$\text{Desk}_1, \ \text{Desk}_2, \ \text{Desk}_3, \text{Desk}_4, \text{Desk}_5.$$

In this example, each of the numbers $1, 2, 3, 4$, and 5 is playing the role of an ordinal number; each ordinal specifies the location of one of the desks. For instance, the number 4 specifies Desk_4. In this example the numbers 1–5 are called the *indices* of the arrangement or ordering.

Sometimes, an arrangement involves so many objects, there are not enough natural numbers (or whole numbers) to specify them all. This is certainly the case for the stages in the build-up of the universe V: After the whole numbers have been used up to specify the stages $V_0, V_1, V_2, \ldots$, more indices are needed to specify the stages that come after these; these new ordinals that come after the whole numbers are called *infinite ordinals*. The first few of these are denoted:

$$\omega, \omega + 1, \omega + 2, \ldots.$$

Therefore, the first stages of V can be written:

$$V_0, V_1, V_2, \ldots, V_\omega, V_{\omega+1}, V_{\omega+2}, \ldots.$$

An important property that all ordinals share with the set $\mathbb{W}$ of whole numbers is given by the following proposition:

> **Proposition** (Well-Ordered Property of Ordinals). *Every nonempty set of ordinals has a smallest element.*

This property is familiar in the context of the set of whole numbers: Given any nonempty set of whole numbers, one can always find its smallest element. For instance, the set of all even numbers bigger than 5 is the set:

$$\{6, 8, 10, 12, \ldots\}$$

and obviously it has a smallest element, namely 6. This same property holds for ordinals.

Another basic but important property of ordinals is that, because of the way they are defined, for all ordinals β and γ, we have that $\beta < \gamma$ if and only if $\beta \in \gamma$. This is obvious for finite ordinals (recalling from the text the set-based definition of the whole numbers): Consider the numbers 2 and 3. By definition

$$2 = \{0, 1\} \text{ and } 3 = \{0, 1, 2\}.$$

Here it is clear that $2 < 3$ and $2 \in 3$. This pattern holds for all pairs of ordinals (finite or infinite).

Finally, we remark that the collection of all ordinals is denoted ON.

A.4. **The Stages of V, Well-Foundedness of Sets, and How Sets Are "Born."** As indicated in the main text, the definition of the universe V and its stages is as follows:

$$\begin{aligned} V_\alpha &= \begin{cases} \mathcal{P}(V_\beta) & \text{if } \alpha \text{ is the immediate successor of } \beta \ (\alpha = \beta + 1) \\ \bigcup_{\beta<\alpha} V_\beta & \text{if } \alpha \text{ has no immediate predecessor.} \end{cases} \\ V &= \bigcup_{\alpha \in \mathrm{ON}} V_\alpha. \end{aligned}$$

The definition tells us that for each stage V_α, the next stage is obtained by collecting together all the subsets of V_α and thereby forming a new, larger set—the power set $\mathcal{P}(V_\alpha)$ of V_α—which is denoted $V_{\alpha+1}$. One writes $\alpha + 1$ to indicate the ordinal number that comes after α; it is the immediate successor of α (just as 4 is the immediate successor of 3). Such an ordinal is called a *successor ordinal* and $V_{\alpha+1}$ is called a *successor stage*.

For ordinals α that have no immediate predecessor (called *limit ordinals*), like ω, the definition tells us that to obtain V_α, we simply collect together all the sets obtained in the stages so far and form the set V_α with these sets. These are called *limit stages*. For such α, no new sets are obtained in the formation of V_α. For instance, ω is a limit ordinal and so V_ω is a limit stage. The stage V_ω contains only those sets that have appeared previously, in one of the stages $V_0, V_1, \ldots, V_n, \ldots$.

The definition makes it clear that, in the build-up of the stages, when a set x appears for the first time in the unfoldment of stages, it does so at a successor stage (since new sets don't arise at limit stages).

We consider an example of the way in which sets first appear in V. We consider the set $\{1\}$. Recall that by definition, $1 = \{\emptyset\}$, so $\{1\} = \{\{\emptyset\}\}$. Recall also the first

four stages in the build-up of V:

$$\begin{aligned} V_0 &= \emptyset \\ V_1 &= \mathcal{P}(V_0) = \{\emptyset\} \\ V_2 &= \mathcal{P}(V_1) = \{\emptyset, \{\emptyset\}\} \\ V_3 &= \mathcal{P}(V_2) = \big\{\emptyset, \{\emptyset\}, \{\{\emptyset\}\}, \{\emptyset, \{\emptyset\}\}\big\} \end{aligned} \tag{A.2}$$

Notice that $1 = \{\emptyset\}$ occurs for the first time in V_2, and so $\{1\}$ is a subset of V_2. Then in the next stage V_3 (which is composed of all the subsets of V_2), the set $\{1\}$ occurs as an element: The set $\{1\}$ is "born" in V_3. One says that the rank of $\{1\}$ is 2—since it occurs as a subset for the first time at stage V_2. In general we have:

> **Definition** (Rank). The *rank* of a set x is the smallest ordinal α for which $x \subseteq V_\alpha$. For this α, one writes $\text{rank}(x) = \alpha$.

This simple example also illustrates an important point: a set is always made up of other sets that occur in earlier stages. As a consequence, we have the following:

> **Propostion** (Rank of Elements). *For each nonempty set $x \in V$ and each $y \in x$, we have* $\text{rank}(y) < \text{rank}(x)$.

A consequence of this proposition is the fact that infinite descending chains of sets

$$\cdots \in y_n \in y_{n-1} \in \cdots \in y_2 \in y_1 \in y_0$$

do not occur in V. Here is a proof: Suppose such an infinite descending chain does exist. Let $\alpha_n = \text{rank}(y_n)$ for each $n \in \mathbb{W}$. Then we have

$$\cdots < \alpha_n < \alpha_{n-1} < \cdots \alpha_2 < \alpha_1 < \alpha_0.$$

In other words, we have a set $\{\alpha_n \mid n \in \mathbb{W}\}$ of ordinals with no smallest element, and this is impossible.

> **Proposition** (No Infinite Descending Chains). *For each $x \in V$, there is no infinite descending chain of sets in V of the form*
>
> $$\cdots \in y_n \in y_{n-1} \in \cdots \in y_2 \in y_1 \in y_0 = x.$$

As mentioned in the text, the rank of a set x in V can be computed by the following formula:

$$\text{rank}(x) = \sup\{\text{rank}(y) + 1 \mid y \in x\}.$$

This is a *recursive* definition. Regarding the notation "sup" seen here: The *supremum* of a set Y of ordinals, denoted $\sup(Y)$, is the smallest ordinal that is greater than or equal to each ordinal in Y.

As an example, we use this formula to compute the rank of the set $x = \{0, 2\}$. It can be shown that, for every ordinal α (finite or infinite), $\text{rank}(\alpha) = \alpha$. In particular, as can be checked in the display (A.2), $\text{rank}(0) = 0, \text{rank}(1) = 1$, and $\text{rank}(2) = 2$. Therefore, when $x = \{0, 2\}$, we compute:

$$\text{rank}(x) = \sup\{\text{rank}(y)+1 \mid y \in x\} = \sup\{\text{rank}(0)+1, \text{rank}(2)+1\} = \sup\{1, 3\} = 3.$$

A.5. **Large Cardinals.** Large cardinals are cardinals (discussed in the main text) whose properties are so strong, it is impossible to prove from ZFC that they exist at all. Nevertheless, as mentioned in the main text, large cardinals appear as key elements of solutions to a variety of research problems.

What ZFC *can* prove about large cardinals is that they lie somewhere in the list of infinite cardinals (mentioned in the main text):

$$\omega_0, \omega_1, \omega_2, \ldots$$

but it is impossible to pinpoint their locations on this list.

Some indication of the difficulty in locating a large cardinal on this list is provided by the fact that every large cardinal is equal to its own index: If ω_α is a large cardinal, then $\alpha = \omega_\alpha$.

More fundamentally, though, large cardinals cannot be proven to exist on the basis of the ZFC axioms for the following reason: The logician Kurt Gödel established that, if the ZFC axioms are consistent (that is, they are not self-contradictory), then ZFC itself cannot *prove* this consistency. The reasoning behind this result is the following; If ZFC *could* prove that a particular cardinal κ is large, it would follow that the stage[28] V_κ is rich enough to be considered a universe of sets—all the ZFC axioms hold in this mini-universe. And this means that ZFC must be consistent. But Gödel's theorem says that ZFC cannot prove its own consistency (unless ZFC is inconsistent)—so ZFC can never prove a cardinal is large.

A.6. **Historical Perspective Concerning Elementary Embeddings of the Universe.** As discussed in the main text, the intuition that the wholeness represented by the universe V should, like pure consciousness itself, have unmanifest dynamics on the basis of which sets (including large cardinals) emerge is realized in the concept of a nontrivial elementary embedding $j : V \to V$ of the universe.[29] Such an embedding, as it acts on sets in V, preserves all (first-order) operations and relationships. An example from the text illustrates this point: If x, Y, and Z are sets in V and $x \in Y$ and the size of Y is less than the size of Z, then j preserves these facts: $j(x) \in j(Y)$ and the size of $j(Y)$ is less than the size of $j(Z)$.

Existence of such an embedding j was once believed to be inconsistent with ZFC. The author [2] carried out a careful study of this claim. First of all, the assertion "there exists a nontrivial elementary embedding from V to V" cannot be stated in the formal language of ZFC; so, technically speaking, one cannot prove that such an embedding leads to inconsistency on the basis of ZFC alone. However, it was shown in a stronger set theory (Morse-Kelley set theory) that existence of such an embedding does lead to inconsistency. But this stronger set theory imposes additional properties on any supposed elementary embedding j of this kind, and [2] showed that inconsistency was a result of the combination of elementarity and

[28]In a few special cases, the model of ZFC obtained from κ is L_κ rather than V_κ, where $L_0, L_1, L_2, \ldots$ are stages, like the stages of V, for building another universe of sets called the *constructible universe* L:

$$L = \bigcup_{\alpha \in \mathrm{ON}} L_\alpha.$$

[29]Such a j is an example of a *function*; the reader may want to review the points about functions in Section A.1.

these other properties. To analyze the situation more carefully, a simple extension of ZFC was proposed in which a unary function symbol **j** is added to the language, intended to represent an elementary embedding in a model of the theory; if one does not assume that ZFC axioms should hold true for formulas in which **j** occurs, this theory is equivalent in strength to ZFC. This framework makes it possible to specify exactly which of the usual ZFC axioms are to be considered valid for these new **j**-formulas. It was shown in [3] that varying the choice of axioms that are allowed for **j**-formulas leads to a variety of set theories, most of which are *not* inconsistent.

When the property of *Amenability* is required of j, the potential for inconsistency is avoided but the new theory is strong enough to account for all the usual large cardinals. Amenability asserts that this j must satisfy the property that, for every set x, the restriction of j to x (denoted $j \restriction x$ and defined by $(j \restriction x)(y) = j(y)$ for all $y \in x$) must belong to V. The new axiom (more precisely, the axiom *schema*) is called the *Wholeness Axiom* or WA:[30]

WA= "there is a nontrivial elementary embedding $j : V \to V$"+Amenability.

The elementary embedding j given by WA is called the *WA-embedding*. From the Wholeness Axiom, it can be shown that all the usual large cardinal properties hold true of the first cardinal that is moved by j (in other words, if κ is least for which $j(\kappa) \neq \kappa$, then κ exhibits all the traditional large cardinal properties).

There is evidence [6] that the theory ZFC + WA is currently the best candidate for a strengthening of set theory from which all traditional large cardinals can be derived, and it is known to be "consistent" in the sense that a model of the theory ZFC + WA is derivable from an I_3-embedding.

The significant point for the present article concerning this history is that existence of such a j can be formalized to avoid historical concerns about inconsistency and to exhibit sufficient strength to derive large cardinals.

A.7. **Laver Sequences As Blueprint.** As mentioned in the text, from a nontrivial elementary embedding $j : V \to V$ (which satisfies Amenability—see the previous section), one may obtain a *Laver sequence*[31] $f : \kappa \to V_\kappa$, using which every set in V can be located. And there is a dual f^{op} of f that returns each x to its source κ. To state these points more rigorously in this Appendix, we define an extension $\ell : V_\kappa \to V_\kappa$ of f and also a dual ℓ^{op} of ℓ. These new versions, like the versions discussed in the main text, are able to capture every set in V and to return each set to its source κ. We outline the ideas behind the proofs of these points—full details appear in [5].

We describe how to obtain a Laver sequence f from a WA-embedding $j : V \to V$. As observed in the main text, j gives rise to a class of somewhat weaker elementary embeddings of the form $i : V_\eta \to V_\xi$ (all with critical point κ) which are called

[30] In the literature, what we are calling here "WA" is usually denoted WA_0; the Wholeness Axiom as it is defined in [2] and [3] is somewhat stronger than WA_0.

[31] Laver sequences were first discovered by R. Laver (see [9]) as a consequence of the existence of a particular type of cardinal called *supercompact*. Later it was discovered that similar Laver sequences are derivable from other kinds of large cardinals. Corazza ([2]) showed that, assuming the Wholeness Axiom, Laver sequences can be shown to exist for many other kinds of large cardinals as well. In the present article, Laver sequences for *extendible cardinals* are discussed; they are called here *extendible Laver sequences*.

extendible embeddings. We will define $f : \kappa \to V_\kappa$ so that it has the *Laver property with respect to* κ—that is, so that for every $x \in V$, there is an extendible embedding i for which $x = i(f)(\kappa)$.

To make sense of this Laver property, we first recall from Section A.3 that every ordinal is itself a *set* whose elements are those ordinals that precede it in the ordering of ordinals. So when we say that the domain of this function f is the ordinal κ, we mean that f acts on the ordinals that are smaller than κ.

Note that we cannot expect to be able to apply such an f to the cardinal κ itself since it is not true that κ is in the domain of f (that is, $\kappa \notin \kappa$). However, κ *is* in the domain of the function $i(f)$ (where i is the appropriately selected extendible embedding). This is because each extendible embedding with critical point κ sends κ to a bigger cardinal $i(\kappa)$. Therefore, by the elementarity of i, since f has domain κ, we have that $i(f)$ has domain $i(\kappa)$, and one of the elements of $i(\kappa)$ is κ. So the formula tells us that when $i(f)$ is applied to κ, it outputs the set x.

We can now define f, using these extendible embeddings: We first build a formula $\phi(g, x)$ that asserts that g is *not* Laver, with witness x, as follows: Let $\psi(\eta, \zeta, i, \alpha)$ be a formula that states formally "$i : V_\eta \to V_\zeta$ is an elementary embedding with critical point α." We let $\phi(g, x)$ be the following formula:

$$\exists \alpha \Big[\, g : \alpha \to V_\alpha \, \wedge \, \forall \eta \forall \zeta \, \forall i \, [(\psi(\eta, \zeta, i, \alpha) \, \wedge \, \mathrm{rank}(x) < \eta < i(\alpha) < \zeta) \\ \to i(g)(\alpha) \neq x] \Big].$$

Now we define $f : \kappa \to V_\kappa$ by

$$f(\alpha) = \begin{cases} \emptyset & \text{if } f \restriction \alpha \text{ is Laver at } \alpha \text{ or } \alpha \text{ is not a cardinal,} \\ x & \text{otherwise, where } x \text{ satisfies } \phi(f \restriction \alpha, x). \end{cases}$$

We wish to show that this definition of f yields the desired result. Although the definition of f seems complicated, the first line in the definition of f is perhaps more understandable, and it provides the key to prove that f has the Laver property. The first line suggests the possibility that for some cardinals α, $f \restriction \alpha$[32] has the Laver property with respect to α. The method of proof to demonstrate that f is Laver is to show that, for "nearly all" $\alpha < \kappa$, $f \restriction \alpha$ does indeed have the Laver property at α.

Technically, what it means to say that, for "nearly all" α, $f \restriction \alpha$ has the Laver property is that the set of all such α is a *big* subset of κ. More precisely, use j to define a set D as follows:

$$D = \{X \subseteq \kappa \mid \kappa \in j(X)\}.$$

The set D has several properties that capture the idea that D consists of the *big* subsets of κ. In the proof, one shows that the set

$$S = \{\alpha < \kappa \mid f \restriction \alpha \text{ has the Laver property at } \alpha\}$$

is a member of D. This means $\kappa \in j(S)$.

[32]Recall that for a function $f : \kappa \to V_\kappa$, the restriction $f \restriction \alpha$ (for any $\alpha < \kappa$) is the function having domain α that takes each $\beta < \alpha$ to $j(\beta)$.

If we compute $j(S)$ we obtain:

$$j(S) = \{\alpha < j(\kappa) \mid j(f) \restriction \alpha \text{ has the Laver property at } \alpha\}.$$

Since $\kappa \in j(S)$, this implies that $j(f) \restriction \kappa$ has the Laver property at κ. A straightforward proof shows that $f = j(f) \restriction \kappa$; therefore, f has the Laver property at κ.

As mentioned earlier, to achieve full rigor we need to define an extension of f to V_κ, which we will denote ℓ; we will have $\ell : V_\kappa \to V_\kappa$. Once we have ℓ, we will be able to define its dual ℓ^{op} on V_κ, which will return each set (in $V - V_\kappa$) to its source (namely, κ). Here are the definitions:

$$\ell(x) = \begin{cases} f(x) & \text{if } x \text{ is an ordinal} < \kappa, \\ x & \text{otherwise.} \end{cases}$$

$$\ell^{\text{op}}(x) = \begin{cases} \alpha & \text{if } \alpha \text{ is the least ordinal for which } \ell(\alpha) = x, \text{ if there is one,} \\ y & \text{otherwise, where } y \text{ is an arbitrary set for which } \ell(y) = x. \end{cases}$$

As is shown in [5], these new versions accomplish the intended goal:

(i) For every x, there is an extendible embedding i such that $i(\ell)(\kappa) = x$

(ii) For every x not in V_κ, there is an extendible embedding i such that $i(\ell^{\text{op}})(x) = \kappa$.

We outline the logic for this here. Let $x \in V$ and let i be an extendible embedding such that $i(f)(\kappa) = x$ (such an i exists since f has the Laver property).

Using the definition of ℓ and elementarity of i, we have that, for all $z \in \text{dom } i(\ell) = V_{i(\kappa)}$,

$$i(\ell)(z) = \begin{cases} i(f)(z) & \text{if } z \text{ is an ordinal} < i(\kappa), \\ z & \text{otherwise.} \end{cases}$$

Applying $i(\ell)$ to κ, it follows that

$$i(\ell)(\kappa) = i(f)(\kappa) = x.$$

We have proven (i).

The argument for (ii) is similar. Using the definition of ℓ^{op} and elementarity of i, we have, for all $z \in \text{dom } i = V_{i(\kappa)}$,

$$i(\ell^{\text{op}})(z) = \begin{cases} \alpha & \text{if } \alpha \text{ is the least ordinal for which } i(\ell)(\alpha) = z, \text{ if there is one,} \\ y & \text{otherwise, where } y \text{ is an arbitrary set for which } i(\ell)(y) = z. \end{cases}$$

We observe that, for all $\alpha < \kappa$ and all extendible embeddings $i : V_\eta \to V_\xi$ with critical point κ,

$$i(f)(\alpha) \in V_\kappa.$$

The proof is based on the fact that for all such i, $i(z) = z$ whenever $z \in V_\kappa$ (we observed earlier that this fact holds for the WA-embedding j; the same reasoning proves it for any extendible embedding i with critical point κ). Since $f : \kappa \to V_\kappa$, it follows that, for each $\alpha < \kappa$ and each $t \in V_\kappa$, if $f(\alpha) = t$ then, by elementarity of i, since

$$f \text{ maps } \alpha \text{ to } t$$

it follows that

$$i(f) \text{ maps } i(\alpha) \text{ to } i(t),$$

and since $i(\alpha) = \alpha$ and $i(t) = t$ we have

$$i(f)(\alpha) = t.$$

On the other hand, since $t \in V_\kappa$, $i(f(\alpha)) = i(t) = t$. It follows that

$$i(f)(\alpha) = i(f(\alpha)). \tag{A.3}$$

Since $t = f(\alpha)$ and $t \in V_\kappa$, we have shown that $i(f)(\alpha) = t \in V_\kappa$.

Now we can show that (ii) holds. If $x \in V - V_\kappa$ and i is such that $i(\ell)(\kappa) = x$ then $i(\ell^{\text{op}})(x)$ is the least ordinal α for which $i(\ell)(\alpha) = x$. By our observation above, if $\alpha < \kappa$, then $i(\ell)(\alpha) = i(f)(\alpha) \in V_\kappa$ and therefore not equal to x. Thus κ is the least ordinal for which $i(f)(\kappa) = x$ and we conclude that $i(f^{\text{op}})(x) = \kappa$.

As mentioned earlier, the result stated in (ii) no longer holds if we require $x \in V_\kappa$. This is because the least α for which $i(f)(\alpha) = x$ is not κ (though κ does satisfy $i(f)(\kappa) = x$). And this is because of the somewhat surprising fact that, for all $x \in V_\kappa$, x is one of the terms of the sequence f (in fact, each such x occurs κ many times in f). We give the reasoning for this here:

Let $x \in V_\kappa$. Let i be an extendible embedding with $i(f)(\kappa) = x$. Let $U_i = \{X \subseteq \kappa \mid \kappa \in i(X)\}$. It is a standard result that such a set U_i has the property that each of its elements has size κ. Let $X = \{\beta < \kappa \mid f(\beta) = x\}$. Then since $i(x) = x$,

$$i(X) = \{\beta < i(\kappa) \mid i(f)(\beta) = x\}.$$

We observe that $X \in U_i$, since κ clearly belongs to $i(X)$. It follows that there are κ many β for which $f(\beta) = x$. We have shown that each $x \in V_\kappa$ is one of the terms of f (and in fact occurs κ many times in f). The conclusion is that it is never the case that $i(f^{\text{op}})(x) = \kappa$ when $x \in V_\kappa$.

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PRINCIPLES OF CONSCIOUSNESS-BASED EDUCATION APPLIED TO A MATHEMATICS COURSE AT MAHARISHI INTERNATIONAL UNIVERSITY

M. ANNE DOW

Abstract. The purpose of this paper is to demonstrate how Consciousness-Based education is used to teach courses at Maharishi International University. The example is a 2-credit online course I designed and taught for the Physiology Department: Mathematics for Aromatherapy. The mathematical principles governing the topics of aromatherapy taught in the lessons and the pedagogical principles used in teaching the lessons are connected to universal Vedic principles from Maharishi Mahesh Yogi's teaching of Vedic Science.

1. Introduction

At Maharishi International University (MIU), all students, faculty, and staff practice the Transcendental Meditation technique as taught by the Vedic scholar and teacher Maharishi Mahesh Yogi. They do this twice a day for about 20 minutes. Numerous research studies have shown that this simple mental technique relieves stress and anxiety that interfere with learning. It also results in increased self-actualization (Alexander et al., 1991), improved brain functioning (Orme-Johnson & Haynes, 1991), improved academic performance (Kember, 1985), increased intelligence (Tjoa, 1975; Tjoa, 1976), increased speed in solving arithmetic problems accurately (Miskiman, 1976b), increased GPA in university students (Collier, 1976; Heaton & Orme-Johnson, 1976), improved recall ability (Abrams, 1976; Miskiman, 1976a), and more.

During the Transcendental Meditation technique (TM), the active thinking mind spontaneously and effortlessly settles down towards the experience of Transcendental Consciousness, a state of restful alertness in which awareness is only aware of itself. I myself have argued (Dow, 1989) that regular experience of this process of transcending to a deeper wholeness of mind also trains the mind in the crucial intuitive step of problem-solving in mathematics, the spontaneous step of illumination, in which the mind is able to cognize deep connections between its own structures and new information, putting them together into a new whole. This step is also the step of the creative process that makes mathematics a joyful and fulfilling experience, which would explain my experience that students at MIU not only succeed at mathematics but also enjoy it.

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Maharishi International University (1981) quotes Maharishi as pointing out the importance of including the practice of TM in any system of education. "If the age is to rise to invincibility, a fundamental value has to be supplied to the field of education: knowledge of pure consciousness and how to experience it" (p. 23) and "Ideal education is capable of providing the fruits of all knowledge while the tree of knowledge is still growing; the student is fulfilled and grows in the ability of being at home with everything while he is still in the process of learning." (p. 45) In other words, we need to develop the knower as well as the information being taught.

In addition, Maharishi (1994) points out: "An ideal educational system should develop wholeness in the life of the learner. The technologies of my Vedic Science strengthen and integrate the physical, mental, and behavioral aspects of life by developing their common basis, pure consciousness. This creates wholeness of awareness, the expression of a truly integrated life, in which all the material, intellectual, and spiritual aspects of life are beautifully correlated." (p. 145)

John Hagelin (1987), working closely with Maharishi, has identified this field of Transcendental Consciousness with the unified field that physics is currently researching and which gives rise to the entire physical universe we live in. Dr. Tony Nader (2024) has taken this even further, bringing out as a basic axiom Maharishi's teaching that consciousness is all there is. This basic axiom is supported by the Vedic literature. The translation into English is by Maharishi:

ātmaivedaṃ sarvam All this is Atman only. (Chāndogya Upaniṣad 7.25.2)

ahaṃ brahmāsmi I am that. (Bṛhadāraṇyaka Upaniṣad 1.4.10)

tat tvam asi Thou art that. (Chāndogya Upaniṣad 6.8.7)

prajñānam brahma And that is consciousness. (Aitareya Upaniṣad 3.1.3)

sarvaṃ khalv idaṃ brahma All this is Brahm—Totality, wholeness. (Chāndogya Upaniṣad 3.14.1)

neha nānāsti kincana Manifest diversity is unmanifest—there is nothing else. (Kaṭha Upaniṣad 2.1.11)

tat sṛṣṭvā tad evānuprāviśat Having created it, the Creator entered into it. (Taittirīya Upaniṣad 2.6.1)

It is also a universal principle recognized by many other people:

"...within man is the soul of the whole; the wise silence; the universal beauty, to which every part and particle is equally related; the eternal ONE." (Emerson, 1841)

Bucke (1901) quotes Plotinus, the 3rd century Greek philosopher, as saying in the *Enneads:* "This region of truth is not to be investigated as a thing external to us ... It is within us. ... Consciousness, therefore, is the sole basis of certainty. ..." (p. 122)

To develop a student's consciousness at MIU we do more than have them practice the Transcendental Meditation technique. This world view, in which we and everything we experience in the mental and physical world around us belong to one undivided wholeness moving within itself according to fixed natural laws described in the Vedic literature, and which can be experienced in its wholeness in higher

states of consciousness, provides a common unified framework of principles governing every discipline and area of life. For example, Maharishi has located one such principle in a verse of the Rig Veda:

> ṛicho akṣare parame vyoman
> yasmin devā adhi viśve niṣeduḥ
> yas tan na veda kim ṛcā kariṣyati
> ya it tad vidus ta ime sam āsate
> (Ṛk Veda Saṃhitā 1.164.39)

This he has translated as:

> "The verses of the Veda exist in the collapse of fullness (the kshara of 'A') in the transcendental field, self-referral consciousness, the Self,
> In which reside all the Devas, the impulses of creative intelligence, the laws of nature responsible for the whole manifest universe.
> He whose awareness is not open to this field, what can the verses [of the Veda] accomplish for him?
> Those who know this level of reality are established in evenness, wholeness of life."

In every discipline, it is possible to locate a collapse of fullness at the foundation of the discipline that is essential for its functioning. We will describe such a collapse of fullness in the Richo Akshare Chart for Algebraic Notation section 8 of this paper.

Einstein (1950) has also described the unity of all disciplines: "All religions, arts, and sciences are branches of the same tree. All these aspirations are directed toward ennobling man's life, lifting it from the sphere of mere physical existence, leading the individual toward freedom." (p. 9)

At MIU, we connect the principles of each discipline to the unifying principles of Maharishi's Vedic Science. In this paper, I will describe the ways in which I incorporate Maharishi's Vedic Science into an online mathematics course that I teach. I'll describe the purpose and implementation of Main Points Charts, Unity Charts, the summary exercise done at the end of a class, the Unified Field Chart for this course, and the Unified Field Chart for all of Mathematics. I will also describe how the structure of lectures and the course itself reflect universal pedagogical principles that come from such Vedic principles. Using these modalities ensures that our students, while learning the topics of the course, never lose sight of the place of the topics in all of mathematics, in all of knowledge, and in the principles governing their own consciousness. This, together with their practice of the Transcendental Meditation technique, ensures that students do not view mathematics as something outside themselves, remote from them, as something they cannot master. They begin to view it as an integral part of themselves and how their own consciousness works and how all of creation works.

Maharishi (Maharishi International University, 1981) reassures us that presenting a course in this way should not interfere with the mathematics we are teaching. "The strength of education lies in its innovative nature; yet the basis of traditional education must be maintained. This means our innovative techniques should not disturb what we are teaching. The basic traditional values must be maintained.

Whatever our methods of teaching, the content of knowledge has to be maintained." (p. 53)

The structure of this paper is as follows. In section 2, we describe how the structure of the course as a whole reflects Vedic principles. In section 3, we discuss the Unified Field Chart for Mathematics showing how all of mathematics springs from the field of Transcendental Consciousness. In section 4, we see an example of a Unified Field Chart for the course Mathematics for Aromatherapy. In section 5, we describe how the structure of each lesson comes from Vedic principles. In section 6, we provide an example of how each topic in each lesson is connected to a universal Vedic principle in the Main Points Chart of the lesson. In section 7, we see in the Unity Chart how the main idea of each lesson is connected to all of mathematics and to ever-deepening levels of the student's own consciousness. In section 8, we present the Richo Akshare Chart for Algebraic Notation, since algebraic notation is fundamental to the course Mathematics for Aromatherapy. The final section 9 presents some overall conclusions.

2. Structure of the Course

A major Vedic principle governing course structure at MIU is this: to be useful, complete, and fulfilling, knowledge should always be connected to its source in the transcendental field. That is, knowledge is best understood and retained when the parts of knowledge are seen in the context of the whole. This principle applied to individual life itself is seen in the Vedic literature in the verse

> *ānandād hy eva khalv imāni bhūtāni jāyante*
> *ānandena jātāni jīvanti*
> *ānandaṃ prayanty abhisaṃviśanti*
> Out of bliss these beings are born,
> In bliss they are sustained,
> And to bliss they go and merge again.
> (Taittirīya Upaniṣad 3.6.1)

Transcendental Consciousness is that transcendental wholeness (Brahman) that gives rise to and contains all of creation. It is also described as an infinite field of bliss, and experience of it is blissful:

> *brahma-saṃsparśam atyantaṃ sukham*
> Contact with Brahman is infinite joy.
> (Bhagavad Gītā 6.28)
>
> *ānandamayo 'bhyāsāt*
> Brahman becomes blissful through practice.
> (Brahma Sūtra 1.1.12)

We can equate "bliss" in this verse to experience of that transcendental wholeness. We structure courses at MIU in parallel with this principle: we start the course with wholeness, maintain the connection to wholeness throughout the course while examining all the parts of the course, and end with wholeness.

The course structure also follows the principle that analysis and synthesis together are the structure of knowledge. A saying that has come down to us in various forms is *anvaya vyati rekabhyam nishprapancham prapanchyate,* which Maharishi

translates as "By virtue of analysis and synthesis, the indivisible Unity is realized in the world of diversity." This may be seen as a variant of a verse in Shankaracharya's Commentary on the Bhagavad Gita 13.13. *tathā hi sampradāyavidām vacanam—adhyāropāpavādāl prapañcam prapañcyate iti.*

In analysis, we break the whole into parts that we analyze. In synthesis, we put parts together into a whole. For example, by meditating twice a day, one experiences the unified wholeness in one's own Transcendental Consciousness, then comes out into the parts of everyday life and schoolwork, then comes back to the wholeness in the next meditation, and so on. For on-campus students, practice of Transcendental Meditation is incorporated into every class at the end of the morning session and at the end of the afternoon session.

Regarding the knowledge of mathematics itself, each lesson is structured so that students experience the wholeness of the lesson at the beginning, then experience the parts of the lesson one after the other and then bring these parts back into an even greater wholeness at the end. In addition, the course as a whole has students experience the wholeness of the course in the first day, then learn the parts of the course day by day over the block, then bring the parts of the course back into a very much larger wholeness at the end with review and a project or exam.

Starting the course with wholeness means that on the first day we first place the course in the wholeness of mathematics and place mathematics in the whole of knowledge including the transcendental field by means of the Unified Field Chart for Mathematics (see section 3). At this point we also give appropriate quotes from Maharishi and famous mathematicians about the importance and usefulness of mathematics in connecting us to the laws governing the world around us. We also place the topics of the course in the wholeness of the course in the Unified Field Chart for the particular course, in this case Mathematics for Aromatherapy (see section 4). Next, we go over the structure, timeline, and content of the course in the Course Overview Chart. Then we present the expected outcomes of the course in a Student Learning Chart in order to make clear to the students how this course is going to benefit them. As well as providing a wholeness for the course, this last chart also reflects the universal principle that knowledge is for action, action is for achievement, and achievement brings fulfillment.

Maintaining the connection to wholeness throughout the course while learning the parts of the course is accomplished by having a Main Points Chart for every lesson listing the important points of the lesson and connecting each important point to a corresponding universal principle from Maharishi's Vedic Science. See section 6 for an example. Also, each assignment has a concluding summary question, bringing the student back to the wholeness of the lesson. At the very end of each lesson, the Unity Chart for that lesson brings all the topics of the main points together again into a wholeness and relates that wholeness to deeper and deeper levels of the student's own consciousness. See section 7 for an example.

The student's attention swings from the mathematical surface level of the mind to the transcendental level, from parts to wholeness, over and over. This is in addition to twice-daily practice of TM, which allows the mind to swing from the concrete level of everyday life and study to the completely abstract transcendental field at the source of all life and back, from the parts of life to the wholeness of life and

back, over and over again. Both these types of swings, one on the intellectual level and the other on the experiential level, help structure higher states of consciousness, in which Transcendental Consciousness is maintained in awareness spontaneously along with waking, dreaming, and deep sleep.

Furthermore, mathematics courses at MIU are usually driven by concrete physical applications that we model with mathematics. This provides another swing from concrete to abstract, from phenomena in the world around us to the wholeness of a mathematical equation.

Ending the course with wholeness is accomplished by having a review and either a project applying all the topics to a real-life problem or an exam. Students also view a short talk by Maharishi at the end of the course discussing principles governing consciousness. The students then write a short essay relating a principle Maharishi speaks about to a basic principle or theme of the course or to their own experience of the course. For example, I may show a talk in which Maharishi explains that during the experience of transcending during TM, one's mind experiences simpler and simpler forms of awareness, and eventually arrives at one's simplest form of awareness, Transcendental Consciousness. The student may relate this idea to solving an equation by transcending the more complicated forms of an equation through simpler and simpler forms to arrive the simplest form of the equation, which gives the answer $x = 7$, say.

3. The Unified Field Chart for Mathematics

At the beginning and end of the course, and then briefly at the end of each lesson, we locate the mathematics that goes into Math for Aromatherapy in the Unified Field Chart for Mathematics (see Figure 1).

This chart covers all of mathematics and its relationship to the world and our own consciousness at a single glance. Maharishi designed this chart to show how the experiential knowledge we gain through the Transcendental Meditation program (the cone on the left) and the objective knowledge we gain through studying mathematics (the chart of mathematics on the right) have a common basis, namely our own Transcendental Consciousness.

In the part of the chart involving mathematics specifically, we move from simple and more basic levels to more complex levels as we move up the chart. Abstract set theory is the most basic level of mathematics and so appears at the bottom. The basic concepts, processes, and objects are next, then the theories of mathematics, which support applications to the sciences, arts, economics, engineering, medicine, and so on, which in turn support careers and national life at the very top.

All these topics, everything in the chart, arise from human intelligence, which has its basis in pure intelligence, Transcendental Consciousness, our own Self, at the very bottom of the chart, which we experience and enliven during practice of Transcendental Meditation, depicted by the cone of transcending on the left.

The arrangement of three columns going from left to right in the mathematical part of the chart has its rationale from the Vedic principle that all knowledge consists of a knower, a process of knowing, and an object of knowledge. For example, near the bottom of the chart, the basic concepts, processes, and objects we study in mathematics are related to the knower, the process of knowing, and the objects

of knowing, respectively. At this level of the chart, we can see the topics of our Mathematics for Aromatherapy course, from left to right:

- The fundamental concepts of arithmetic, namely counting, number, arithmetical operation, estimation, proportion, rate of change, formula, and equation at the left;
- The fundamental processes of logic, critical thinking, and mathematical justification in the middle;
- The fundamental mathematical objects such as the numbers themselves, the plus and minus signs, straight lines, and equations on the right.

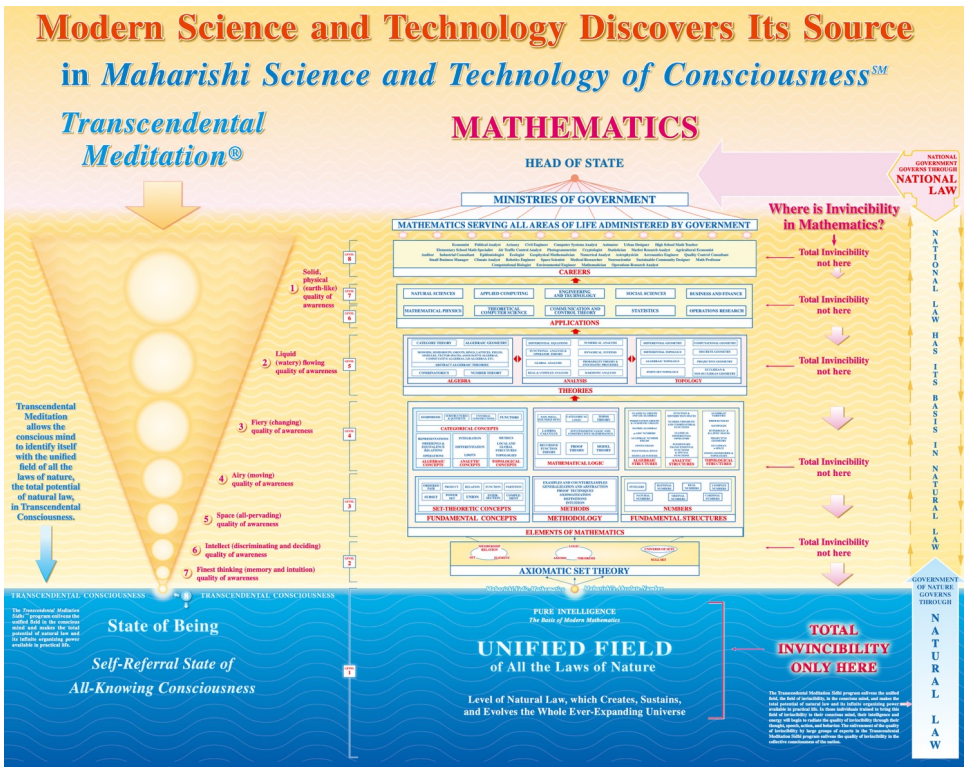

FIGURE 1. Excerpt from the Unified Field Chart for Mathematics (Reproduced with permission of Maharishi Vedic University Ltd.)

At the theory level of the chart, we depict that there are really only three basic components of all mathematical theories:

- Algebra on the left, which is about operations, such as addition, multiplication, subtraction, division, and their rules and which we associate with the knower, since it is the basic math that first occurred to us historically as knowers of natural phenomena.
- Analysis in the middle box, which is about change and starts with functions and calculus and which we associate with process and transformation.

- Geometry, in the topology box on the right, which is about shapes and forms and seems more concrete than the rather abstract algebra and analysis.

The vast majority of all mathematical theories are based on combinations of these three components.

Using the fundamental concepts, methods, and objects at the lower level, we develop the simple formulas and procedures used to create safe and effective blends for aromatherapy. These formulas and procedures reside in the theory boxes.

In turn, we apply these formulas and procedures to numerical situations that come up in aromatherapy contexts. These applications reside in the applications boxes at the top of the chart.

4. The Unified Field Chart for the course Mathematics for Aromatherapy

The lessons in the course Mathematics for Aromatherapy, together with related universal principles, are as follows.

Theme 1. Numbers *Well-begun is half done*

1.1. Whole numbers *Self-interacting dynamics of wholeness*

1.2. Fractions *Harnessing the orderliness of nature*

1.3. Decimal numbers and percents *Using the infinite to describe the finite*

Theme 2. Measurement *Connecting the concrete with the abstract*

2.1. Units *Connecting the concrete world with abstract thought*

2.2. Unit conversion using the unit-analysis method *Locating non-change within change*

Theme 3. Solving equations *Transcending to the simplest form of awareness*

3.1. Preparing to solve equations *Mastering the abstract in order to handle the concrete*

3.2. Solving equations *Maintaining balance*

Theme 4. Bringing it all together *Connecting parts to wholeness*

4.1. Final project *Knowledge is for action, action is for achievement, and achievement brings fulfillment*

These topics allow the students to solve the problems involved in creating blends. What is a blend? It consists of a small amount of essential oil in a large amount of a carrier oil. For example, a recipe may call for a 30 ml blend with a 3% concentration of lavender oil in a coconut oil carrier. A typical problem would be to determine how much lavender oil and how much coconut oil to measure out and combine to make a total of 30 ml. This can get complicated. Recipes may be given in percentages, fractions, or parts. A single recipe may include several essential oils in one carrier oil. Amounts may appear in various units: drops, milliliters, cubic centimeters, grams, teaspoons.

Students need to calculate and measure very carefully in order to ensure both effectiveness and safety. Too little essential oil means the blend may not produce the desired effect. Too much means the person may have an adverse reaction.

Problems involving recipes require understanding the arithmetic of various forms of numbers and also how measurement works. Being able to construct and solve equations comes in when one has made a mistake and put in too much or too little

essential oil, and one wants to correct the situation without throwing the whole blend away and starting again. Also, equations come in when combining blends with two different concentrations of the same essential oil to make a third blend with a required concentration. The same mathematics is needed to calculate yield when distilling plant material to extract the essential oil.

These topics are arranged in the Unified Field Chart for the course showing how they are developed and related. You should imagine the Unified Field Chart for Mathematics with the mathematics part replaced by the aromatherapy chart (see Figure 2).

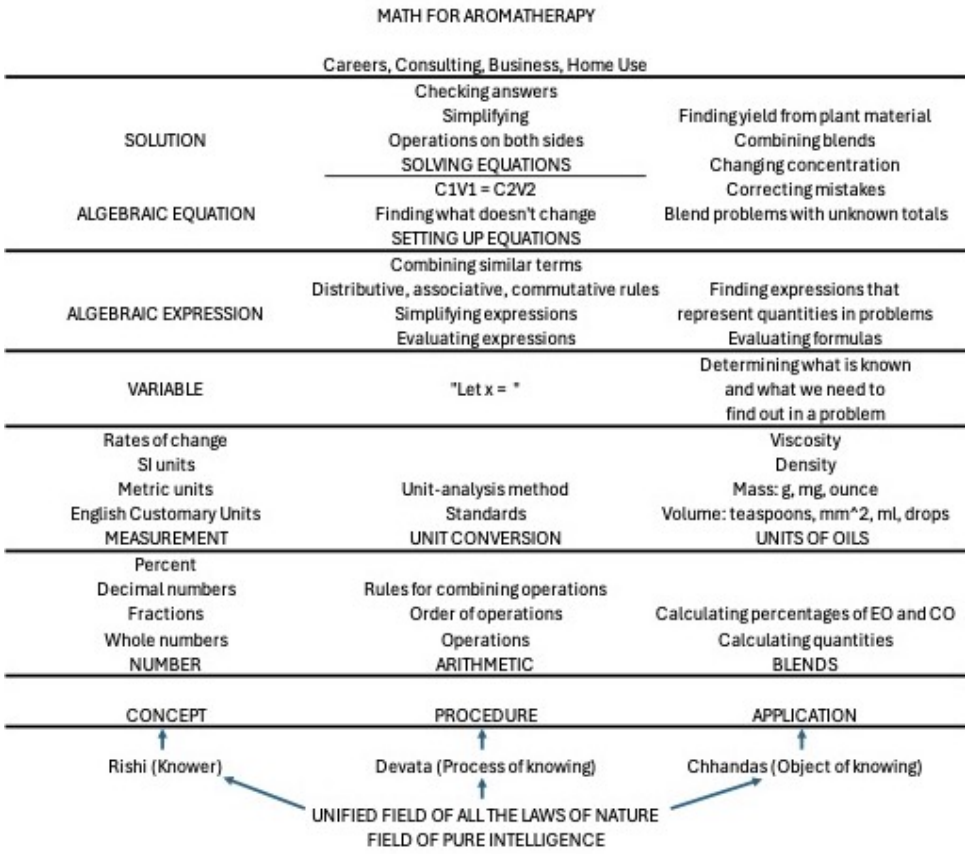

FIGURE 2. Excerpt from the Unified Field Chart for the course Mathematics for Aromatherapy

The topics of the course are arranged in three columns: the left column contains the abstract mathematical concepts needed, the middle column contains the mathematical tools we develop in the course, and the right column contains the applications of the tools to aromatherapy problems. The three columns come, as

before, from the Vedic principle that all knowledge consists of a knower, a process of knowing, and an object of knowledge. In this chart, we think of the knower as embodying the abstract mathematical concepts, the process of knowing as the methods of solving mathematical problems, and the object of knowing as particular applications of these concepts and methods to solving problems in aromatherapy.

The chart then proceeds from the simplest ideas at the bottom to more and more complex ideas as we go up the chart, culminating in being able to apply the tools developed in the course to model mathematically and solve problems in an aromatherapy career or in using aromatherapy in one's own home.

The foundation and driving force in all parts of this chart is your own intelligence, which has its basis in pure intelligence, pure consciousness, your own Self, depicted at the bottom of the chart, and which is experienced and enlivened during Transcendental Meditation. There at the bottom is pure Transcendental Consciousness, the Self, alone by itself, in which the knower is pure consciousness, the process of knowing is pure consciousness, and the object is pure consciousness. This is known in Maharishi's Vedic Science as the three-in-one structure of the field of pure consciousness. It is both one and three at the same time, giving rise to a vibration of infinite frequency that creates the universe we live in.

5. Structure of a Lesson

A lesson usually consists of a lecture and/or workshop in which the topics related to each main point in the Main Points Chart are covered one after the other. When the topic covered in one main point has been completed, we read the main point and its corresponding Vedic principle. Then students work on an exercise or activity to integrate the mathematical ideas presented in that section of the lecture and one or two present their results to the class. Then we go on to the topic of the next main point, and so on. See section 6 for an example of a Main Points Chart.

This structure allows swings between concrete and abstract, between parts and whole, but it also reflects the principle: knowledge is for action, action is for achievement, achievement brings fulfillment.

The lecture or workshop is followed by an assignment. The last question on the assignment is a summary exercise:

(a) Write a sentence explaining the most important mathematical principle you learned in this lesson.
(b) Write down a principle governing consciousness that illuminates your important mathematical principle and explain the connection clearly.
(c) Draw a picture, figure, or sketch illustrating your principle.

This summary question provides a mathematical wholeness and a Vedic wholeness that comes from the student's own understanding of the lesson. It helps the student integrate the knowledge of the lesson into their knowledge base. In addition, sketching a figure at the end ensures that this understanding is not just words and equations, but also has a visual component, thus enlivening more of the whole brain.

At the end of the lesson, we present the Unity Chart that brings all the topics of the main points together again into wholeness and relates that wholeness to deeper and deeper levels of the student's own consciousness. See section 7 for an example of a Unity Chart.

6. The Main Points Chart

Each Main Points Chart includes a mathematical title for the lesson and a subtitle relating the mathematical title to a universal principle from Maharishi Vedic Science.

MATH142 Mathematics for Aromatherapy
Handling the Concrete by Enlivening Its Source in the Abstract

Module 3
SOLVING EQUATIONS
Knowledge is for Action

LESSON 7
SOLVING EQUATIONS
Transcending to Our Simplest Form of Awareness Locates the Field of Non-Change Within

Wholeness of the Lesson

Equations involving unknown quantities arise in answering several types of questions in aromatherapy by locating what does not change. Solving an equation for an unknown quantity x is a matter of undoing the operations on x that led to the equation in the first place. We perform addition, subtraction, multiplication, division to both sides of the equation and simplify the results step by step until x appears by itself on one side of the equal sign and its value appears on the other side of the equal sign.
Consciousness: In Transcendental Meditation we experience finer and finer levels of the mantra until we reach our simplest state of awareness, pure consciousness, the Self.

Main Points

1. Solving an equation for a single variable is simply a matter of performing the same arithmetic actions on both sides of the equation, with the aim of undoing the operations that were done on x, and simplifying, over and over again, until x appears by itself on one side of the equation. Then the other side is the solution.
   Consciousness: Transcendental Meditation allows the active thinking mind to spontaneously experience simpler and simpler levels of awareness until one is left alone with one's own pure consciousness.

2. If we have put too much essential oil in a blend, we can fix it by adding more carrier oil. In order to find out how much carrier oil to add, we locate what doesn't change, here the amount of essential oil, and thereby create an equation, whose solution is the amount of carrier oil to add.
   Consciousness: Handling the field of change in the world we live in is best done by contacting the field of non-change within.

3. If we have put too little essential oil in a blend, we can fix it by adding more essential oil. To find out how much essential oil to add, we locate what doesn't change, here the amount of carrier oil, and thereby create an equation, whose solution is the amount of essential oil to add.
   Consciousness: Handling the field of change in the world we live in is best done by contacting the field of non-change within.

4. If we have two blends on hand with different concentrations of the same essential oil, we can create a new blend of any desired concentration between the two concentrations of the oil we have on hand. We find the amounts of each to mix together by creating and solving two equations in two unknown quantities.
   Consciousness: The nature of life is to grow towards more and more knowledge, success, and happiness. That growth is enhanced by experiencing the unchanging field of consciousness within us that is already there.

Figure 3. Main Points Chart for Lesson 7 of MATH142 Mathematics for Aromatherapy

The title and subtitle are followed by a statement giving the main idea to be covered in that lesson. This statement is connected to a short elaboration of the Vedic principle referred to in the subtitle of the lesson. The body of the chart consists of three or four main points to be covered in the lesson. Each of these main points provides both a mathematical main point and a related Vedic principle governing consciousness. Each section of the lecture concludes with its main point and corresponding connection to consciousness. Figure 3 is an example from Mathematics for Aromatherapy. Notice how each title and each point is related to a principle governing consciousness. Maharishi's Vedic Science teaches that the transcendental field at the source of creation is non-changing. It moves within itself to create the universe we live in, but the universe actually remains within the wholeness that is the transcendental field. In that sense, the transcendental field is non-changing, eternal. See the Vedic quotations in section 1 above for support from the Vedic literature for this principle.

In point 2, for example, the equation we create is based on the fact that the amount of essential oil remains the same before and after we correct the blend, since we will need to add more carrier oil. In point 3, the equation is based on the fact that the amount of carrier oil remains the same, since we will need to add more essential oil. An equation is created when we locate what does not change. At the deepest level, all change in creation has its source and foundation in the non-changing field of pure consciousness experienced within us during Transcendental Meditation.

7. The Unity Chart

The Unity Chart for a lesson consists of 5 points. See Figure 4.

- The first point summarizes the lecture from the point of view of the mathematics we covered in the lesson.
- The second point views the lesson from a deeper level or principle of mathematics, so that the student sees the lesson in the light of all of mathematics, not just as an isolated topic.
- The third point moves to the student's own consciousness. Hence the dotted line as we enter that realm. It describes an experience or principle of the student's own Transcendental Consciousness that is related to the topic of the lesson: so that they see the lesson as relating to them, not just as information.
- The fourth point relates the topic of the lesson to the warming up of Transcendental Consciousness, the first impulses of natural law, which are enlivened by regular experience of Transcendental Consciousness and are responsible for the entire manifest universe.
- The fifth and final point expresses qualities of Unity Consciousness that are related to or support the theme of the lesson. It says how one would experience the culmination of the principles introduced in the lesson, when one achieves the highest state of consciousness, Unity Consciousness. It gives a vision of the wholeness of all of life, flavored with the theme of the lesson, of how the theme of the lesson rises to its supreme value in our own awareness when we reach Unity Consciousness.

UNITY CHART
Connecting the Parts of Knowledge with the Wholeness of Knowledge
Solving Equations

1. Many problems in aromatherapy can be solved by locating what does not change and thereby creating an equation that we can solve to get the answer.
2. Solving equations easily and successfully depends on using algebra skillfully to simplify the equation while maintaining the balance of the two sides of the equation.

3. **Transcendental Consciousness** is experience of the non-changing field of pure consciousness within.
4. **Impulses within the transcendental field:** Regular experience of this non-changing field brings orderliness to the field of change in everyday life.
5. **Wholeness moving within itself:** In Unity Consciousness, we experience all change in the world around us as waves on the ocean of the non-changing transcendental field within us. The infinite organizing power of that field becomes available to us to accomplish anything we wish.

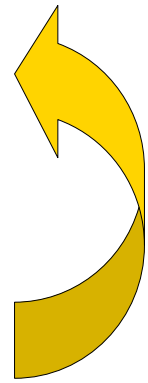

FIGURE 4. The Unity Chart for Lesson 7 of the course Mathematics for Aromatherapy

- And finally, the gold arrow on the right of the Unity Chart expresses the fact that all knowledge of specific principles and topics of mathematics depends upon and has its source in pure consciousness. In practical terms, the more expanded and developed our own consciousness, the more deeply we appreciate and understand the principles and theme of the lesson. We appreciate and understand that the principles of the lesson are universal and have their source in our own awareness.

8. The Richo Akshare Chart

As mentioned earlier, Maharishi has located a fundamental principle in Ṛk Veda I.164.39 that applies to all disciplines by locating the "collapse of fullness" within the Transcendental Field that is essential to the success of that discipline. Since algebraic notation is fundamental to solving problems in aromatherapy, we identify the collapse of all possible values into a single symbol or letter as essential for the success of algebra. Here is the verse of Ṛk Veda together with the interpretation in terms of algebraic notation.

ONE VERSE OF ṚK VEDA (I.164.39) CONTAINS ALL THEORIES OF MATHEMATICS: ALGEBRAIC NOTATION

richo ak-kshare

The verses of the Veda exist in the collapse of fullness (the kshara of 'A')...

Algebraic notation uses symbolic expressions *(richā)* to formulate the language of mathematics. An algebraic symbol represents all possible mathematical objects

that satisfy a specific collection of properties. Thus, algebraic notation collapses the wholeness of all possibilities into a symbol (kshara of 'A').

parame vyoman

...in the transcendental field, self-referral consciousness, the Self...

The collapse of mathematical objects into a symbol takes place in the awareness of the mathematician. It is the mind of the mathematician that connects the appropriate mathematical meaning to a specific symbol. For example, mathematicians may associate the symbol x with all possible solutions of an equation.

yasmin deva

...in which reside all the Devas, the impulses of creative intelligence, the laws of nature...

Mathematicians create basic rules of syntax (Devas) to formulate the symbolic representations of mathematical statements. These rules allow mathematical ideas to be expressed in a compact and precise form.

adhi vishwe nisheduh

...responsible for the whole manifest universe.

Using these rules of syntax, the whole range of mathematical knowledge can be expressed in algebraic notation; this range includes the axioms of all mathematical theories, all the theorems of these theories, and all the proofs that deduce these theorems from the axioms.

yastanna veda

He whose awareness is not open to this field...

If one is unable to maintain the connection between a symbol and its meaning, one is restricted to the use of ordinary language or to the formal manipulation of meaningless symbols.

kim richa karishyati

...what can the verses [of the Veda] accomplish for him?

This makes it difficult to express abstract knowledge precisely or meaningfully and makes the process of mathematical proof extremely cumbersome. Without this understanding of algebraic notation, the process of generalization by abstraction in modern mathematics would be obstructed, and none of the theories of modern mathematics could have been developed.

ya it tad vidus

Those who know this level of reality...

Mathematicians who maintain a fluid connection between algebraic symbols and their meaning are able to use algebraic notation to capture the essence of mathematical concepts in a compact, easily handled form.

ta ime samasate

...are established in evenness, wholeness of life.

This enables them to clarify crucial issues in a mathematical problem and to reason on more abstract and powerful levels. Algebraic notation provides a common symbolic language for the exact expression, development, and unification of all mathematical knowledge. The ability to communicate mathematical knowledge in

a precise and unambiguous way, using this universal notation, allows for unrestricted progress in all areas of mathematics and its applications.

9. Conclusion

The purpose of Consciousness-Based education at MIU is to connect the individual topics and concepts of a course to the broader, deeper reality of mathematics or of any discipline, and to the broadest, deepest reality: the Self of the student, which they experience twice daily during their practice of the Transcendental Meditation technique. This is important because both experience and intellectual understanding are necessary to gain and retain knowledge at a deep level. But also, connecting the limited parts of a course to their unbounded source in the unlimited field of the students' own potential ensures that students never lose sight of who they really are. They never lose their connection with the basis of knowledge as they explore the full range of knowledge in their classes.

Maharishi has inspired the faculty of MIU to connect every wave of knowledge to the Self, so that the students develop the feeling of being familiar and intimate with everything and everyone. This leads to greater self-confidence and self-sufficiency, which gives them a balanced and integrated personality. Maharishi (Maharishi Vedic University, 1986) has explained that both experience of the transcendental field of pure consciousness and connecting the subject of study with that transcendental field are necessary for education to fulfill its goals:

"All the universities are simply hovering on the surface of knowledge. If, along with the study of each subject, the experience [of pure consciousness] is taught to the students, then they will be able to fathom the deeper levels of that subject, and the whole range of that subject will be studied properly. When the two extremities of that subject—the gross, expanded value and the transcendental value—are connected, then the field of that subject will be complete, and the study of that subject will bring something real and useful in life." (p. 269)

In this paper, I have explained how we bring education to a higher level at MIU in the form of Consciousness-Based education, which is based on principles cognized and lived by the seers of the Vedic tradition down through the ages, and which have been revived in their completeness and purity by Maharishi Mahesh Yogi in this current age.

My experience teaching mathematics at MIU has kept me here for 40 years because I see the fruits of this approach in my students. I see the fulfillment of Maharishi's promise that this approach will lead to graduates who enjoy and succeed and find fulfillment in life. Many research studies have shown improvements in every area of life, including providing the clarity of thinking that leads to students' success in their studies, simply through the practice of the Transcendental Meditation technique. In my experience, adding the modalities of Consciousness-Based education to university education at MIU has expanded my students' awareness so that they enjoy all disciplines as part of themselves. They become happy, successful, peaceful individuals who relate to the whole world as their family and will accomplish something highly useful and fulfilling in their lives.

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VEDIC MATHEMATICS AND COGNITIVE EASE: REDUCING MATHEMATICAL ANXIETY THROUGH PATTERN-BASED THINKING

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Abstract. Vedic Mathematics, popularized by Bharati Krishna Tirthaji, offers a computational framework that emphasizes pattern recognition, flexibility, and mental visualization rather than rote memorization. This paper explores how Vedic Mathematics may reduce mathematical anxiety by lowering cognitive load and promoting cognitive ease. Drawing upon Cognitive Load Theory, research on working memory, and studies on mathematics anxiety, the paper argues that Vedic methods function as psychologically effective cognitive shortcuts that simplify arithmetic and encourage conceptual fluency. By transforming mathematics into a more intuitive and confidence-building experience, Vedic Mathematics may help students develop positive mathematical identities and increase participation in STEM-related disciplines. The paper further examines the broader educational and social implications of reducing mathematical anxiety, arguing that approaches that make mathematics more accessible may contribute to greater scientific literacy, creativity, and human development.

1. Introduction

Mathematics occupies a central position in modern civilization. Scientific progress, engineering innovation, medicine, economics, and technological development all depend heavily upon mathematical thinking. Despite its importance, mathematics remains one of the most feared academic subjects and one of the most misunderstood throughout history. In fact, as cited in Kline (1962), as far back as AD 400, St. Augustine, Bishop of Hippo in Africa and one of the great fathers of Christianity, said that "The good Christian should beware of mathematicians and all those who make empty prophecies. The danger already exists that the mathematicians have made a covenant with the devil to darken the spirit and to confine man in the bonds of Hell." As indicated also by Kline, "the vastly increased importance of mathematics in our time makes it all the more imperative that the modern person know something of the nature and role of mathematics." However, many associate mathematics with stress, embarrassment, frustration, and failure rather than creativity and discovery.

Researchers have shown that mathematical anxiety significantly affects performance, motivation, and self-confidence (Tobias, 1993). Students who repeatedly experience frustration in mathematics frequently begin to avoid quantitative disciplines altogether. As a consequence, many capable learners exclude themselves

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from careers in science, technology, engineering, and mathematics. Boaler (2016) argues that rigid instructional practices and excessive emphasis on memorization often reinforce these negative perceptions.

Within this context, alternative pedagogical approaches that promote cognitive ease and conceptual understanding deserve careful consideration. One such approach is Vedic Mathematics, a collection of computational techniques described by Bharati Krishna Tirthaji (1965). Unlike conventional arithmetic instruction, Vedic Mathematics encourages learners to identify numerical patterns, symmetries, and structural relationships. This paper explores how Vedic Mathematics may reduce mathematical anxiety, increase engagement with mathematics, and contribute to broader educational and societal development.

2. Mathematical Anxiety and Cognitive Load

Mathematical anxiety is commonly defined as a feeling of tension, fear, or apprehension that interferes with mathematical performance (Ashcraft & Krause, 2007). Importantly, mathematical anxiety is not necessarily associated with low intelligence or lack of ability. Many highly capable students experience severe emotional stress when confronted with mathematical tasks.

According to Baddeley (1992), working memory possesses only limited capacity for simultaneously storing and processing information. Traditional arithmetic procedures often require learners to manage multiple intermediate steps, remember carrying operations, align symbols correctly, and maintain procedural accuracy under time pressure. When cognitive demand exceeds working-memory capacity, confusion and anxiety frequently emerge.

Sweller's (1988) Cognitive Load Theory distinguishes among intrinsic, germane, and extraneous cognitive load. Extraneous cognitive load results from instructional methods that impose unnecessary mental effort. Many conventional approaches to arithmetic unintentionally generate excessive extraneous load because students are asked to memorize procedures before understanding numerical relationships.

Vedic Mathematics may help alleviate this problem by reorganizing arithmetic into shorter, pattern-based operations. Instead of focusing exclusively on procedural survival, learners can devote greater mental energy to conceptual understanding and confidence building.

3. Pattern Recognition and Cognitive Ease

A central feature of Vedic Mathematics is its emphasis on pattern recognition. Human cognition naturally seeks patterns because pattern-based processing reduces mental effort and improves retention. When learners identify structure and regularity within mathematics, they often experience what psychologists describe as cognitive ease. Vedic Mathematics is a collective set of sixteen mathematical formulae (sutras). Each formula deals with a different branch of mathematics. As D. Bathia (2005) points out, "Complex mathematical questions which otherwise take numerous steps to solve can be solved with the help of a few steps and in some cases without any intermediate steps at all! And these systems are so simple that even people with an average knowledge of mathematics can easily understand them."

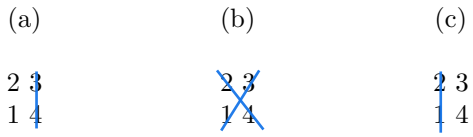
For example, the sutra "All from Nine and the Last from Ten" simplifies subtraction from powers of ten through numerical complements. For example, consider

$$1000 - 357 = 643$$

Instead of performing traditional borrowing operations, the student computes complements:

$$\begin{aligned} 9 - 3 &= 6 \\ 9 - 5 &= 4 \\ 10 - 7 &= 3 \end{aligned}$$

Similarly, the sutra "Vertically and Crosswise" transforms multiplication into a visually organized set of interactions among digits. These approaches reduce the number of disconnected operations learners must remember, and encourage conceptual understanding. When multiplying numbers such as 23×14 using the conventional method, students often memorize several procedural steps involving carrying and alignment. The Vedic approach instead organizes the multiplication into a sequence of crosswise combinations as shown next:



$$\begin{aligned} &3 \times 4 \\ &(2 \times 4) + (3 \times 1) \\ &2 \times 1 \end{aligned}$$

This structure encourages students to perceive multiplication as an interconnected pattern rather than a mechanical algorithm.

Boaler (2016) argues that learners develop deeper understanding when mathematics is presented conceptually rather than mechanically. Vedic Mathematics supports this perspective by encouraging mental flexibility and numerical intuition. Students who understand why a computational shortcut works often develop stronger confidence and more positive attitudes toward mathematics.

The psychological consequences of cognitive ease are significant. Learners who experience mathematics as manageable and intuitive are less likely to develop fear-based reactions. This emotional transformation may be especially important for students who have previously struggled with traditional instruction.

4. Confidence, Motivation, and Mathematical Identity

Educational psychologists have long recognized the importance of mathematical identity. Students gradually form beliefs about whether they are "good at mathematics," and these beliefs strongly influence motivation and persistence.

Repeated experiences of confusion or failure often produce negative mathematical identities characterized by avoidance and low self-confidence. In contrast, repeated experiences of success can foster curiosity, resilience, and intellectual engagement. Vedic Mathematics may contribute positively to this process because many of its techniques produce rapid and satisfying results.

According to Tobias (1993), reducing anxiety is essential for improving mathematical participation. Students who feel emotionally safe in mathematical environments are more likely to ask questions, explore ideas, and persist through challenges. Learners who become comfortable with mathematics may be more willing to pursue disciplines such as engineering, computer science, economics, statistics, and physics.

In this sense, Vedic Mathematics may function not only as a computational tool but also as a psychological intervention capable of reshaping learners' emotional relationships with mathematics.

5. Educational and Social Implications

The educational implications of reducing mathematical anxiety are substantial. Mathematics functions as a gateway subject for many academic and professional opportunities. Students who avoid mathematics frequently restrict their future career possibilities.

Approaches that make mathematics more intuitive and less intimidating may therefore contribute to broader educational inclusion. Vedic Mathematics may be especially valuable for learners who struggle with procedural memorization but possess strong intuitive or visual reasoning abilities.

The cultural dimension of Vedic Mathematics is also important. Modern mathematics education often emphasizes European mathematical traditions while overlooking contributions from other civilizations. By introducing learners to Indian mathematical thought, educators may promote greater cultural inclusivity and intellectual diversity.

The societal implications may be even broader. In contemporary society, quantitative literacy has become essential for informed citizenship. Public policy debates involving economics, healthcare, climate science, artificial intelligence, and technological ethics increasingly require mathematical understanding.

A population that feels comfortable with mathematics is better equipped to participate critically in democratic decision-making and scientific discourse. Furthermore, societies that cultivate mathematical confidence may produce more innovators, scientists, engineers, and problem-solvers. Approaches that reduce fear and increase intellectual confidence may therefore contribute positively to human development.

6. Limitations and Balanced Perspectives

Despite its potential advantages, Vedic Mathematics should not be idealized or treated as a universal replacement for conventional instruction. Some critics argue that certain techniques are highly specialized and may not generalize easily to advanced mathematical contexts.

Additionally, empirical research specifically examining the long-term effects of Vedic Mathematics on anxiety reduction remains limited. More classroom-based

studies are needed to evaluate retention, transferability, conceptual understanding, and psychological outcomes.

Nevertheless, the broader pedagogical principles underlying Vedic Mathematics—pattern recognition, cognitive efficiency, flexibility, and conceptual understanding—align strongly with contemporary findings in cognitive science and mathematics education (Sweller, 1988; Baddeley, 1992).

7. Conclusion

Vedic Mathematics offers more than a collection of rapid computational shortcuts. It represents a pattern-based philosophy of mathematical thinking that may reduce anxiety, promote cognitive ease, and encourage intellectual curiosity. By emphasizing structure, flexibility, and mental visualization, Vedic methods can reduce unnecessary cognitive burden and help learners experience mathematics as accessible and meaningful.

Students who develop confidence and curiosity in mathematics are more likely to engage with STEM disciplines, participate in quantitative reasoning, and contribute creatively to society. Reducing mathematical anxiety is therefore not merely an educational objective but also a broader social goal connected to empowerment, opportunity, and human flourishing.

While further empirical research is needed, the theoretical connections among Vedic Mathematics, cognitive load reduction, and mathematical confidence suggest considerable pedagogical promise. Educational systems should strive not only to produce correct answers but also to cultivate learners who feel capable, curious, and unafraid to think mathematically.

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International Journal of Mathematics and Consciousness

In recent centuries, scientists have found that many phenomena in nature obey physical laws that can be expressed precisely in the language of mathematics. Their successes have led scientific inquiry beyond the physical world to include what was previously considered metaphysical or non-material. Today, an increasing number of scientists are examining the nature of consciousness and its relationship to the human brain.

While most models of consciousness propose that it is a product of chemical and electrical behavior within the brain, no current theory resolves the so-called "hard problem of consciousness"—how physical processes in the nervous system give rise to subjective experiences such as experiencing, thinking, feeling, analyzing, and creating. At the same time, it is undeniable that without awareness—without consciousness—we cannot think, perceive, dream, or love. On this basis alone, a scientific journal dedicated to exploring the nature of consciousness is timely and appropriate.

While consciousness can be studied within a variety of disciplines, mathematics especially lends itself to examine the relationship between consciousness and physical phenomena. Mathematics is precise and rigorous in its methodology, giving symbolic expression to abstract patterns and relationships. Although developed subjectively, using intuition along with the intellect and logical reasoning, mathematics allows us to make sense of our outer physical universe. Mathematics is the most scientific representation of subjective human intelligence and thought, formalizing how individual human awareness perceives, discriminates, organizes, and expresses itself.

The scientific consideration of consciousness by itself is a formidable challenge, for consciousness is a purely abstract reality. But the study of what we might call "consciousness at work"— how consciousness expresses itself in our daily activity of thinking, analyzing, creating, theorizing, and feeling—is inherently more accessible. For this exploration also, mathematics is the ideal tool, because its ability to express patterns of abstract human awareness helps us make sense of our physical universe. One could in fact argue that mathematics is the most scientifically reliable tool for the exploration of the dynamics of consciousness, for it alone can be seen as the symbolic representation of "consciousness at work."

The International Journal of Mathematics and Consciousness will help to fulfill the need for a forum of research and discussion of consciousness and its expressions. The editors invite mathematicians, scientists, and other thinkers to present their theories of consciousness without restriction to proposed axioms and postulates, with the stipulation only that such theories follow strict logical argumentation and respect proven facts and observations. Articles that use factual or logical counterarguments to challenge commonly believed but not fully established facts and observations are also welcome.